

SETTLEMENT AND SHEAR STRENGTH ANALYSIS OF STONE COLUMN REINFORCED SOFT SOIL

**A Thesis Submitted
in Partial Fulfilment of the Requirements
for the Degree of
MASTER OF TECHNOLOGY**

By
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to the
**DEPARTMENT OF CIVIL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR
March, 1989**

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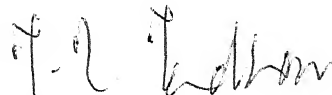
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CERTIFICATE

It is certified that this work 'Settlement and Shear Strength Analysis of Stone Column Reinforced Soft Soil' by Bhupesh Kumar Chandola has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.


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ACKNOWLEDGEMENT

I express my deep sense of gratitude to my thesis supervisor, Dr. M.R.Madhav for his invaluable guidance, suggestions and help at every stage of the preparation of this thesis. It has been a matter of pride and pleasure for me to be associated with him. His criticisms and encouragement have gone a long way in completing this thesis. I am highly indebted to Dr. M.R.Madhav and Dr. P.K.Basudhar for their kind assistance.

The support given by Mr. A.K.Srivastava, Mr.R.P.Trivedi, Mr. Gulab Chand and Mr. Parasuram of the geotechnical engineering laboratory is duly acknowledged.

I am thankful to all my friends, with special mention to Mr. J.P.Sharma, Vijay Prakash, Mohd. Javed, Sanjay Parikh and Chandan Ghosh, who made my stay here a memorable one.

I.I.T.Kanpur

B.K.CHANDOLA

March, 1989

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ABSTRACT

Soil reinforcement is one of the ground improvement techniques, which has been intensively studied and advanced in application in the past. The use of stone columns as a ground improvement technique is of recent origin. In stone column reinforced soft soils, a considerable reduction in settlement, increase in bearing capacity and shear strength has been observed.

For each stone column, there exists a domain of influence and thus in the analysis, the group effect of stone column-soil system is idealized by a unit cell of a given diameter. In this thesis an analytical solution has been developed for the settlement of the stone column treated ground by considering a typical stone column-soil unit. A set of equations are obtained for the settlement analysis of unit cell by considering the axial and radial compatibility between the stone column and the surrounding soil. The shear strength analysis of a unit cell is also presented. From this analysis the equivalent shear strength of a unit cell is determined. Model studies have been carried out in the laboratory. Direct shear tests have been performed on soil, granular material and column-soil unit for various column diameters. The experimental results are compared with the analytical results.

CHAPTER 1

INTRODUCTION

The increased cost of conventional foundations and numerous environmental constraints greatly encourage the in-situ improvements of weak subsoil deposits. To economically develop marginal sites a number of new ground improvement techniques have been recently developed. Some of these techniques are feasible for present use but many require considerable additional research.

Large number of methods have been proposed to increase the bearing capacity and to reduce the settlement. The application of these methods depends on properties such as shear strength, compressibility and permeability of in-situ soil, the depth and thickness of the compressible layer, the type of structure, available time and equipment, local expertise, environmental factors, cost etc.. The basic requirements for the satisfactory performance of a foundation are that the foundation should be safe in shear and the settlement should be within the tolerable limits. For this the fundamental approach should be, to reduce the load on the compressible layer by using light weight material, to transfer the super structure weight to the more competent layer, to replace the compressible material by more competent material, to increase the shear strength and reduce

the compressibility through dewatering and preloading or by stabilization of compressible layers. To achieve these requirements, various ground improvement techniques on soft soil may be enlisted below :

1. Installation of sand drains/wick drains to expediting the rate of settlement by reducing the consolidation time.
2. Dynamic compaction, to compact and consolidate the weaker layers by repeated dropping of a heavy weight on the surface of soil.
3. Lime piles, to increase the shear strength and reduce compressibility.
4. Stone column/Granular piles.
5. Chemical stabilization.
6. Thermal stabilization.
7. Consolidation by preloading and/or vertical drains.
8. Heating and freezing.
9. Use of geotextiles.

Stone columns or granular piles are one of the ground improvement techniques having a proven record of experience. They are ideally suited for improving soft clays and silts and also for loose silty sands. Stone column technique for ground improvement has been in use for almost more than two decades. However the vibrofloatation process for stabilizing deposits of soil was first conceived in the early 1930's.

When the structure is to be constructed on soft or weak and compressible soils, stone column could be the most ideal choice from the point of view of technical feasibility and low

energy utilization. Further the merit of the stone column is the ability to adjust to the applied load and thereby redistribute the load where stress concentration occurs. The stone column system, therefore has an inherent advantage that there is no collapse of stone column because an overloaded stone column automatically gets relieved of the stresses as it deforms.

In stone column construction, usually 15 to 35 percent of the weak soil volume is replaced by stones. Design loads on stone column typically vary from 20 to 50 tons. The presence of the column creates a composite material of lower overall compressibility and higher shear strength than the native soil alone.

Stone columns may be constructed by :

- a. Vibroreplacement (wet) process.
- b. Vibroreplacement (dry) process.
- c. Rammed stone columns.

The installation of stone column is more of an art than an exact science; therefore it requires careful field control and an experienced contractor. The rammed stone columns are constructed either by driving an open or closed end pipe in the ground or boring a hole. A mixture of sand and stones (12-75 mm.diameter) are placed in the hole in increments rammed by using a heavy falling weight.

Subsurface conditions for which stone columns are in general not suited include ; 1) layers of peat, decomposable organics or refuse greater than one to two stone column diameter in thickness, 2) sensitive clays and silts which loose there

strength when vibrated and 3) weak strata not underlain by a competent bearing layer. In special cases even these soils may be improved, but not without extreme care and perhaps great expense. Further the various advantages of stone columns may be enumerated,

1. Installation is relatively simple and involve low energy input or manual labour.
2. Moderate increase in load carrying capacity.
3. Significant reduction in settlement.
4. Being granular and freely draining, consolidation settlement is accelerated and post construction settlement will be minimized.
5. Substantial economies can be achieved compared to other type of deep foundations.
6. In combination with preloading, further increase in bearing capacity and decrease in settlement can be achieved.
7. Stone columns can successivly be installed even in the deposits of very low shear strength upto undrained shear strength of 7kN/sqm .
8. The ground treated with stone columns withstand earthquake forces better and resist liquifaction because the pore water pressure generated due to the cyclic loading, which is the major cause of the liquifaction, can be dissipated as fast (almost) as they are generated. thus the danger of the liquifaction can be averted.

For each specific application, however, stone columns should be carefully compared with the other design alternatives, considering both the advantages and limitations of each method.

CHAPTER 2

LITERATURE REVIEW

Though the technique of reinforcing weak subsoil stratum by stone columns was known to the french engineers in 1830's, the process drew the attention of research workers only during the last two decades. In the Glasgow district the concept of stabilized ground using stone columns was first applied in 1962. The earliest attempt for the design of stone columns was given by Thornburn and Mecvicar (1968). Their design was based on empirical approach.

Thornburn and Mecvicar (1968) proposed a relationship between allowable load on a granular pile and the undrained shear strength, C_u , of the cohesive soil mass surrounding the pile, based on the result of actual load tests on granular piles. A similar relationship between working load and undrained shear strength was proposed by Hughes and Withers (1974).

Greenwood (1970) proposed a method based on passive pressure or plastic failure approach. He used Rankine passive earth pressure coefficients. The load is applied through the granular pile and as the pile material dilates it exerts lateral stresses on the surrounding clay which are resisted by the passive earth pressure.

Vesic (1972) gave the solution to the problem of expansion of spherical and cylindrical cavities in $C-\phi$ soil. The

solution takes into account the effect of volume change in the plastic region.

Datye and Nagaraju (1975) used the cavity expansion approach proposed by Vesic (1972) and computed the limit lateral stress of a granular pile in a soft saturated clay. For different values of rigidity index, Datye and Nagaraju (1981) have computed the ultimate bearing capacity of the granular piles for a range of compressibilities (loose to dense sand deposits).

Baumann and Bauer (1974) describe a method of estimating the settlement of the stone column reinforced soil under a given load condition. They stated that the vibro-replacement method is effective only up to the limit where the sensitivity of clay is less than five.

A series of model tests were carried out at Cambridge University using radiographic technique to determine the actual behaviour of single granular piles in normally consolidated clay by Hughes and Withers (1974).

Hughes *et al.* (1975) presented interesting data on the behaviour of a granular pile subjected to load upto its ultimate capacity. The stone column was constructed by vibroreplacement method for load test and after the test the column was excavated to determine its deformed shape. These deformed dimensions are compared with those observed by Hughes and Withers (1974), in their model tests. Both the deformed shapes are geometrically very similar. They report that the failure shape in upper part is like a bucket resting on cylindrical stem. They suggested bulging as the most likely mode of failure and proposed a method of estimating the load carrying capacity of the column.

Engelhardt and Golding (1975), describe the soil improvement with stone columns in an area of high seismic susceptibility. Direct shear tests were performed in the field to demonstrate the horizontal shear strength developed by a stone column improved subsoil. They have noticed that the shear strength parameters of the combined mass of stone column and native intervening soils are significantly higher than the parameters of the in-situ soils existing prior to the stone column installation and that the resultant shear strength can safely resist horizontal forces induced by ground acceleration.

Balaam et al. (1977), employed both the finite element and finite difference methods for the theoretical prediction of the magnitude and the rate of settlement of soft clays reinforced by granular piles. They used Biot's consolidation equation and diffusion theory to calculate the degree of consolidation settlement. This analysis reveals that the significant reduction in settlement may occur only if the stone columns are closely spaced ($d_e/d \leq 5$) and usually only if the piles are installed to the full depth of the consolidation layer. Estimates of the optimal spacing, diameter and degree of penetration of the piles can readily be made from the results presented.

Seed and Booker (1977), developed design principles for stone columns to stabilize a soil deposit susceptible to liquefaction so that the pore water pressures generated by cyclic loading could be dissipated almost as fast as they are generated.

Rao and Bhandari (1977), report that a rigid skirt provided at the top portion of the column prevents lateral

deformation and thus bulging. It was shown that skirting (up to a depth of about 0.8 meter) increases the ultimate stress intensity by 50% over unskirted pile capacity.

Madhav and Vitkar (1978), considered two dimensional plane stress variation of a granular pile as a granular trench and postulated the general shear failure mechanism. The bearing capacity of a footing on a soil stabilised with a granular trench is determined. The bearing capacity factors have been evaluated and presented in the form of graphs.

Balaam and Booker (1979), proposed an analytical solution using the theory of elasticity for the settlement of a rigid rock on a stone column reinforced soil. In the analysis, the elastoplastic behaviour of the soil and elastoplastic and dilatant behaviour of the pile material are taken into account. They have also given the expression for evaluating the moment and shear distribution across the foundation. The charts for the variation of the stresses in stone column and clay with time were also presented.

Goughnour and Bayuk (1979), considered the behaviour of a single stone column and its surrounding soil as a unit cell. The assumption in this theory is that the pile is linearly elastic, perfectly plastic at failure and incompressible in the plastic state. The time- settlement behaviour of the stress distribution between stone column and soil have been analysed and a correlation has been made with actual field tests.

Datye and Nagaraju (1981), discussed the different types of installation methods of the stone columns highlighting

proper field control and quality control. The testing of stone columns were also discussed by them. The cost effectiveness of various ground improvement techniques were reviewed with particular emphasis to the stone columns.

Madhav (1982), further suggested the improvement in the response of the granular pile reinforced soil by providing rigid (Concrete) plug near the top and reported that if only top 15% of the pile length is rigid, the load carrying capacity doubles and if top 30% length is rigid, it becomes four times.

Datye (1982), proposed an approach wherein a simple model of stone column behaviour, elastic or plastic, can be chosen and varified by load tests. The suggested approach requires close control on installation and choice of suitable spacing.

Van Impe and De Beer (1983), presented a simple way of estimating the improvement of the settlement behaviour of the stone column reinforced soil by considering the stone column to deform at constant volume. The only parameters to be known were the geometry of the pattern of the stone columns, their diameter, the angle of shearing resistance of the stone column material, the oedometer modulus of the soft soil and its poisson's ratio.

Balaam and Booker (1983), described an interaction analysis to predict the load -settlement response of rigid foundation supported by clay stabilized by stone columns. The elastic settlement correction factors are provided. The solution of the proposed finite element analysis are found in the close agreement with field observations at the actual sites.

Runesson et al. (1985), deal with the problem of

consolidation of a clay layer in which vertical partially penetrating drains have been installed. The influence of wall resistance was considered in the analysis.

Mitchell and Timothy (1985), described the performance of the stone column foundation to support a large wastewater treatment plant founded on 15 meter of soft estuarine deposit. 6500 stone columns were installed, out of which 28 single stone columns were tested for load-settlement behaviour. 30 to 40% reduction in the settlement to the values to be expected on unimproved ground were observed. Measured settlement varied from 25-60 mm. whereas a settlement of about 65 mm. was predicted from finite element analysis.

Rao and Ranjan (1985), presented a method of computing the settlement of weak subsoil deposits reinforced by skirted granular piles using the concept of equivalent coefficient of volume compressibility. Full scale in-situ tests on skirted granular piles were carried out and settlement computations from proposed analytical procedure were compared with field values where a good correlation is noted. The usefulness of this method is because of its versatility to accommodate changing subsoil conditions with depth and based on the soil parameters which can be easily estimated. However, Goughnour (1988), has expressed doubt about its effectiveness in soft cohesive subsoil deposits where the pile material and the in-situ soil are assumed to behave as a linearly elastic material.

The soil-column interaction depends on several parameters including the loading process and the loading rate, the replacement factor, the group effect and the partial

consolidation of soil due to radial drainage through the column. Juran and Guermazi (1988), presented a laboratory study that reveals the effect of these parameters on the settlement response of a soft foundation soil, reinforced by compacted stone columns.

Goughnour (1988), has expressed his views that in case of granular piles installed in less compressive material such as silty sand, the bulging phenomenon is insignificant. Granular pile do bulge in loose to medium dense cohesionless soils also.

Gopal Ranjan (1989), synthesizes the current state of the art of stone column construction and design. The various techniques of ground improvement was reviewed with emphasis laid primarily on the stone columns.

CHAPTER 3

ANALYSIS OF UNIT CELL3.1 PART 1 : SETTLEMENT ANALYSIS OF UNIT CELL

3.11 DESCRIPTION OF PROBLEM :

The settlement of rigid foundation supported by soft clays and stabilized by large number of stone columns installed over an extensive area in a regular pattern is considered. In this case, each column and the surrounding soil respond in virtually the same fashion as those adjacent. It follows from the considerations that the settlement of the foundation can be estimated from an analysis of a typical cylindrical unit of stone column and its domain of influence.

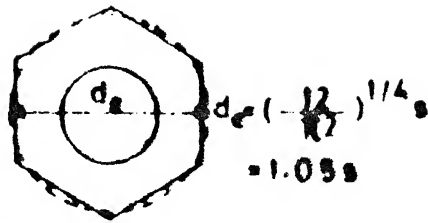
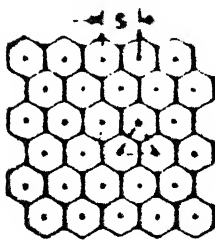
3.12 CONCEPT OF UNIT CELL :

Since large number of stone columns are installed, there must be a regular pattern of their arrangement. The three patterns of stone columns are shown in Fig.3.1. For a triangular arrangement, each column has an hexagonal zone of influence, whereas a square arrangement gives each column a square zone of influence and hexagonal arrangement gives triangular zone of influence. This last pattern is of very limited practical importance. In order to reduce the complexity of the analysis, the zone of influence is approximated by a circle of effective diameter d_0 ; where,

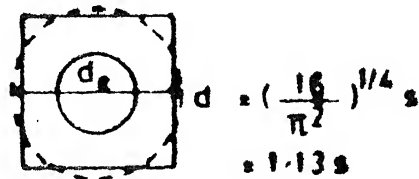
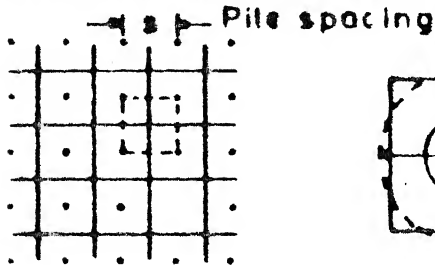
$$d_0 = 1.05 S , \text{ for triangular arrangement}$$

$$d_0 = 1.13 S , \text{ for square arrangement}$$

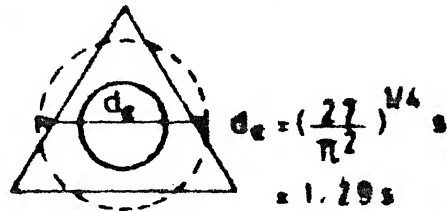
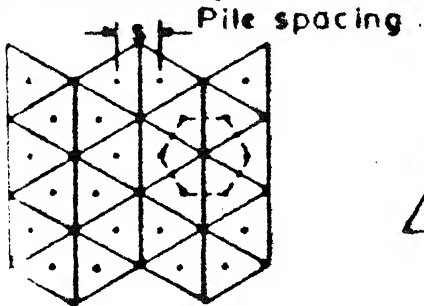
Pile spacing



(a) Triangular arrangement of stone columns



(b) Square arrangement of stone columns



(c) Hexagonal arrangement of stone columns

Fig.3.1 VARIOUS STONE COLUMN ARRANGEMENTS SHOWING THE DOMAIN OF INFLUENCE OF EACH COLUMN.

and $d_0 = 1.29 S$, for hexagonal arrangement

where,

S = spacing between the stone columns

The resulting equivalent cylinder of diameter d_0 , enclosing the surrounding soil and one stone column is known as the unit cell.

Because of the symmetry of the load and geometry, lateral deformations can not occur across the boundaries of unit cell. Also from the symmetry of load and geometry, the shear stress on the outside boundaries of the unit cell must be zero. The distribution of the stresses within the unit cell between the stone column and the surrounding soil is as shown in the Fig.3.2a.

The elastic response of a typical column-soil unit can be calculated using an analytical approach. For this, the stone column and the surrounding soil are assigned Young's moduli, E_p and E_s , Poisson's ratios, ν_p and ν_s respectively. The assumption is that the soil follows a linear stress-strain law but its modulus of deformation varies with depth. Thus, layered or nonhomogeneous profiles are considered.

Although the approximations of the actual domain of an equivalent circular domain has considerably simplified the analysis, an examination of the problem shows that a complete solution still depends upon various dimensionless parameters such as,

$$S = d_0/d_p$$

$$M_r = E_p/E_s$$

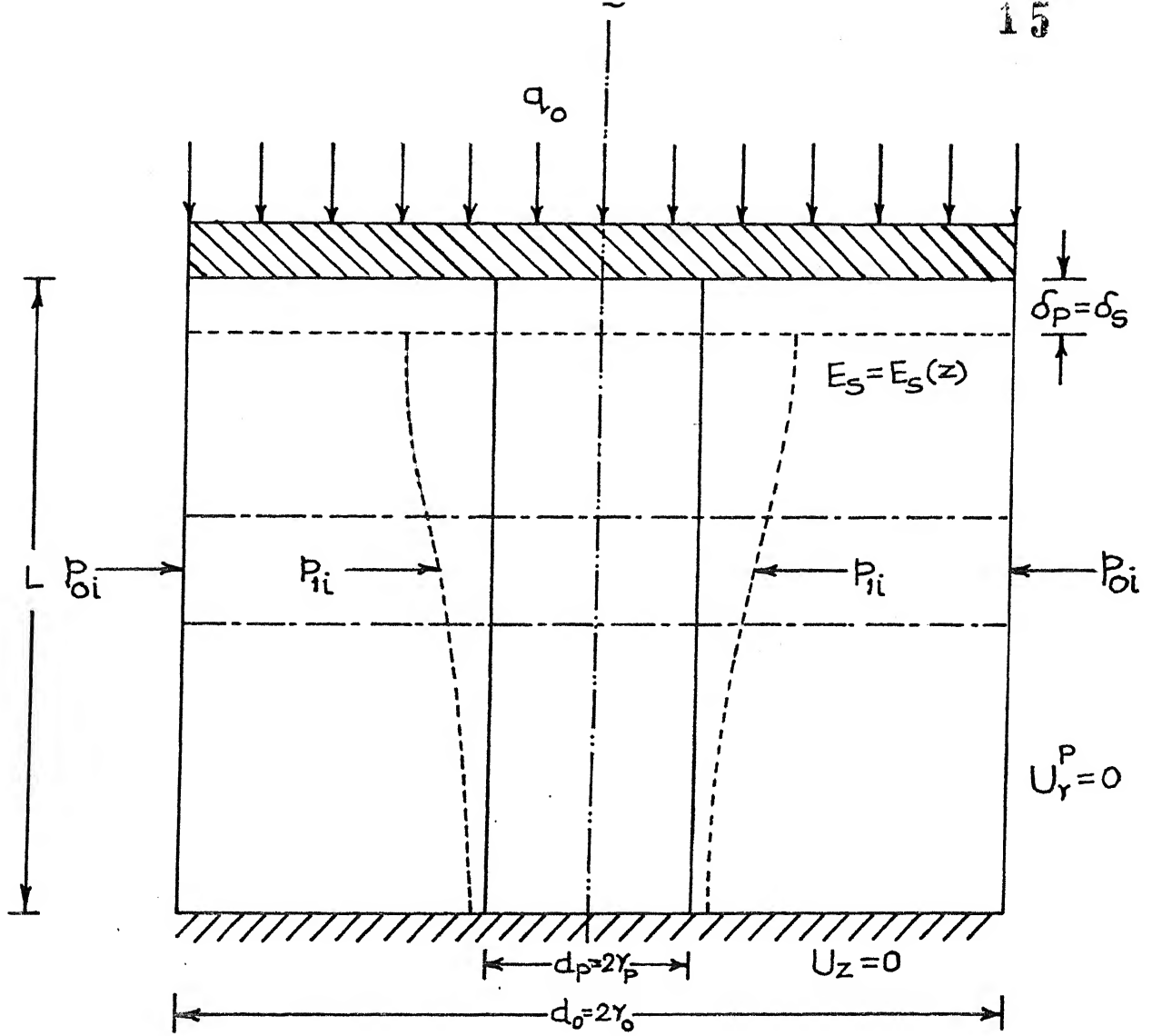
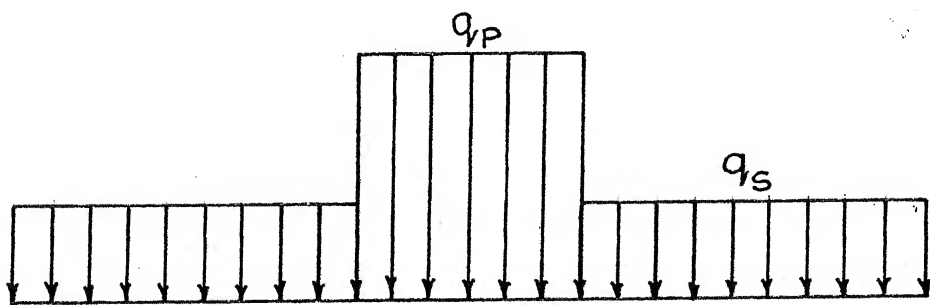


Fig.3.2.a UNIT CELL

Fig.3.2.b NORMAL STRESS DISTRIBUTION BETWEEN
STONE COLUMN AND SOIL.

3.13 METHOD OF ANALYSIS :

Fig.3.2a depicts a typical unit of a stone column-soil system, from either a triangular or square array of stone column of diameter d_p , spacing S and length L . The diameter d_0 of the equivalent zone is a function of the type of arrangement and the spacing. This unit is subjected to a uniform pressure of intensity q_0 . This pressure is distributed between stone column and soil as shown in Fig.3.2b. Fig.3.3a and Fig.3.3b illustrate the normal and radial stress distribution between the surrounding soil and stone column respectively. The radial stresses are varying with the depth due to the nonhomogeneity of the soil.

The various factors which are taken into consideration for the analysis are as follows :

1. The contact stress between the rigid raft and the soil is uniform over the column and uniform over the surrounding soil .
2. The base of the soil-column unit is rigid ie, $U_z = 0$.
3. The outer boundary of the unit cell is smooth and rigid and having no radial displacement ie, $U_r^s = 0$.
4. The vertical displacement is almost uniform over any horizontal plane and varies almost linearly from zero at the base to a maximum value at the surface .
5. The soil follows a linear stress-strain law but with its modulus of deformation varying with the depth ie, $E_s = E_p$, ie, the non-homogeneous or layered profile is considered .
6. The typical soil-column unit is divided into ten number of layers . Each soil layer is treated as a thick cylinder with an internal pressure of p_{1i} and an external pressure of p_{oi} .
7. The stress state in the stone column is triaxial ie, $\sigma_r = \sigma_\theta$.

A set of equations are developed for the

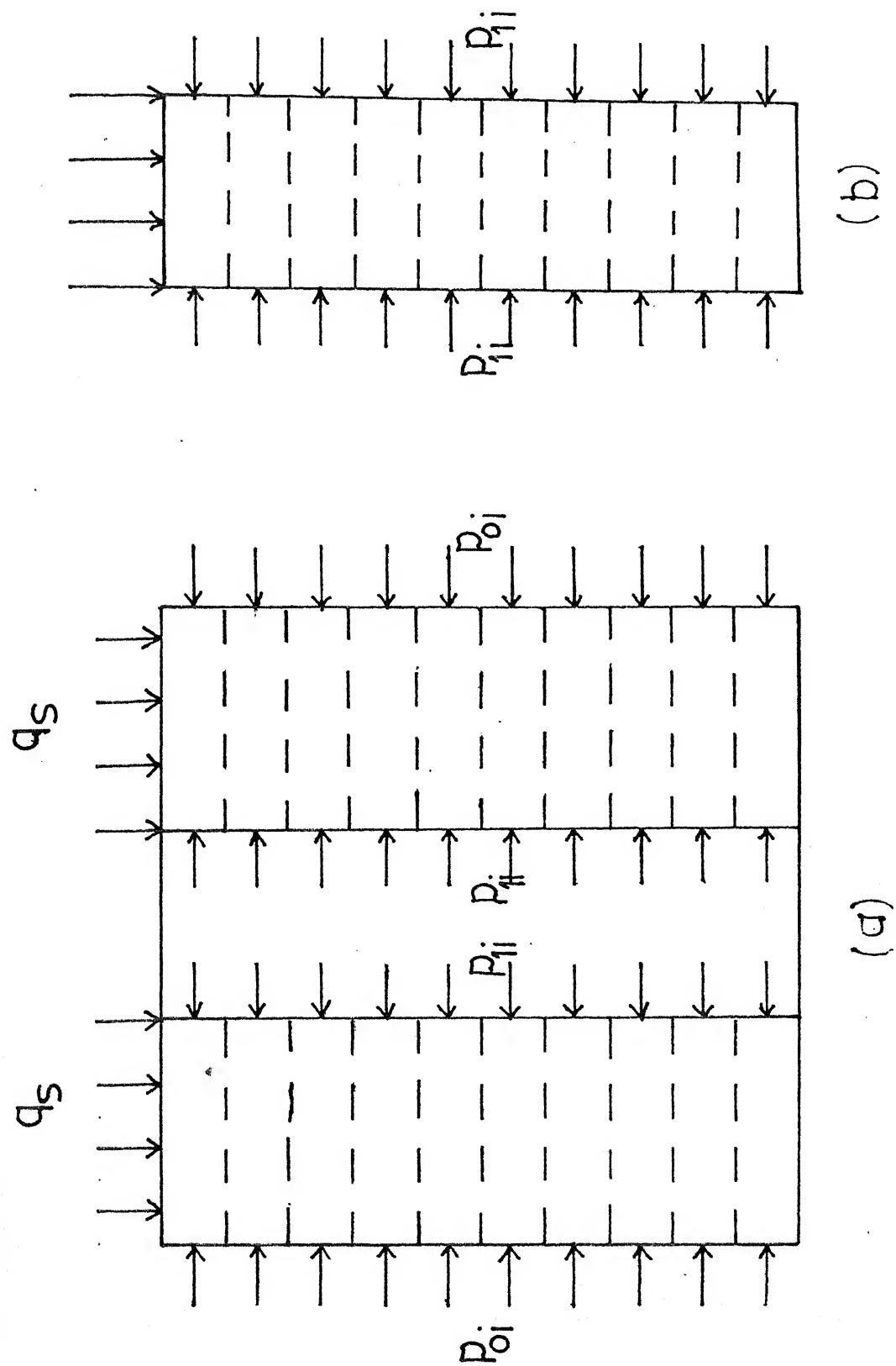


Fig. 3.3.3 · DEFINITION SKETCH

analysis of this cylindrical unit as below ,

a) Equilibrium Requirement :

The applied uniform stress of intensity q_o is shared between the column and soil such that,

$$q_o \cdot \frac{\pi d_o^2}{4} = q_p \cdot \frac{\pi d_p^2}{4} + q_s \cdot \frac{\pi}{4} (d_o^2 - d_p^2) \dots\dots\dots 3.1$$

Normalizing this equation with respect to q_o and d_p , and simplifying ,it becomes,

$$Q_p + Q_s (S^2 - 1) = S^2 \dots\dots\dots 3.2$$

where,

$$Q_p = q_p / q_o$$

$$Q_s = q_s / q_o$$

$$S = d_o / d_p$$

q_o = applied uniform normal stress

q_p = stress on stone column

q_s = stress on surrounding soil

b) No radial displacement at outer boundary :

$$U_r^s(r_o) = 0$$

$$\text{ie, } \epsilon_{\theta i}(r_o) = 0$$

$$\text{or, } r_o / E_{si} [\sigma_{\theta i} - \nu_s (\sigma_{r i} + \sigma_z)] = 0 \dots\dots\dots 3.3$$

(NOTE: The column-soil unit is divided into number of layers and the i^{th} layer is considered . Compression as negative and tension as positive are the considered sign convention.)

At the outer boundary ie at $r = r_o$

$$\sigma_{ri} = - p_{oi} \dots\dots\dots 3.4.a$$

$$\text{and } \sigma_{\theta i} = \frac{2 r_p^2 p_{1i} - p_{oi}(r_o^2 + r_p^2)}{(r_o^2 - r_p^2)} \dots\dots\dots 3.4.b$$

so from above equⁿs,

$$\frac{2 r_p^2 p_{1i}}{(r_o^2 - r_p^2)} - \frac{p_{oi}(r_o^2 + r_p^2)}{(r_o^2 - r_p^2)} + v_s \cdot p_{oi} + v_s \cdot q_s = 0 \dots\dots\dots 3.5.a$$

$$\text{or, } p_{oi} [r_o^2(1-v_s) + r_p^2(1+v_s)] = p_{1i}(2r_p^2) + v_s(r_o^2 - r_p^2)q_s \dots\dots\dots 3.5.b$$

$$p_{oi} = A p_{1i} + B q_s \dots\dots\dots 3.6$$

normalizing it with respect to q_o ,

$$P_{oi} = A P_{1i} + B Q_s \dots\dots\dots 3.7$$

$$\text{where, } P_{oi} = p_{oi} / q_o$$

$$P_{1i} = p_{1i} / q_o$$

$$A = \frac{2}{S^2(1-v_s) + (1+v_s)} \dots\dots\dots 3.8.a$$

$$B = \frac{v_s(S^2)}{S^2(1-v_s) + (1+v_s)} \dots\dots\dots 3.8.b$$

c) Radial Displacement of Surrounding Soil at Inner Boundary =
Radial Displacement of Stone Column :

$$U_r^s(r_p) = U_r^p(r_p)$$

For soil displacement,

$$U_r^s(r_p)^* = \frac{1+\nu_s}{E_{si}} \left[\frac{p_{1i}r_p^2 - p_{oi}r_o^2}{(r_o^2 - r_p^2)} (1-2\nu_s)r_p - \frac{r_o^2 \cdot r_p^2 (p_{oi} - p_{1i})}{(r_o^2 - r_p^2)r_p} \right] - \nu_s \epsilon_z r_p \quad \text{-----} 3.9$$

where,

$$\epsilon_z = \frac{1}{E_{si}} \left[\sigma_z - \nu_s (\sigma_{ri} + \sigma_{\theta i}) \right]$$

$$\epsilon_z = -\frac{q_s}{E_{si}} - \frac{2\nu_s}{E_{si}} \left[\frac{p_{1i}r_p^2 - p_{oi}r_o^2}{(r_o^2 - r_p^2)} \right] \quad \text{-----} 3.10$$

therefore,

$$U_r^s(r_p) = \frac{r_p}{E_{si}} \left[\frac{(1-\nu_s-2\nu_s)(p_{1i}r_o^2 - p_{oi}r_o^2) - (1+\nu_s)^2(p_{oi}r_o^2 - p_{1i}r_o^2)}{(r_o^2 - r_p^2)} \right] + \frac{r_p}{E_{si}} \left[\nu_s \cdot q_s + 2\nu_s^2 \left(\frac{p_{1i}r_p^2 - p_{oi}r_o^2}{(r_o^2 - r_p^2)} \right) \right] \quad \text{-----} 3.11$$

$$U_r^s(r_p) = \frac{r_p}{E_{si}} \left[\nu_s \cdot q_s + \frac{p_{1i}(r_p^2 + r_o^2) - 2p_{oi}r_o^2 + \nu_s p_{1i}(r_o^2 - r_p^2)}{(r_o^2 - r_p^2)} \right] \quad \text{-----} 3.12$$

For stone column, radial displacement @ $r=r_p$,

$$U_r^p(r_p) = \epsilon_{\theta i} \cdot r_p = \frac{r_p}{E_{pi}} \left[\sigma_{\theta i} - \nu_p (\sigma_{ri} + \sigma_z) \right] \quad \text{-----} 3.13$$

since triaxial state of stresses are acting on stone column,

$$\sigma_{ri} = \sigma_{\theta i} = -p_{1i} \quad \text{-----} 3.14$$

* The derivation of this expression is given in APPENDIX A

Equation 3.13 becomes,

$$U_r^p(r_p) = \frac{r_p}{E_{pi}} \left[-p_{1i} v_p (-p_{1i} - q_p) \right] \text{-----3.15.a}$$

or,

$$U_r^p(r_p) = \frac{r_p}{E_{pi}} \left[v_p q_p - p_{1i} (1 - v_p) \right] \text{-----3.15.b}$$

Equating eqⁿ 3.12 and eqⁿ 3.15.b ,

$$\frac{r_p}{E_{si}} \left[v_s q_s + \frac{p_{1i}(r_p^2 + r_o^2) - 2p_{oi}r_o^2 + v_s p_{1i}(r_o^2 - r_p^2)}{(r_o^2 - r_p^2)} \right] = \frac{r_p}{E_{pi}} \left[v_p q_p - p_{1i}(1 - v_p) \right] \text{-----3.16.a}$$

or,

$$p_{oi} (2r_o^2 \cdot E_{pi}) = p_{1i} \left[E_{pi} r_o^2 (1 + v_s) + E_{pi} r_o^2 (1 - v_s) + (1 - v_p) E_{si} (r_o^2 - r_p^2) \right] \\ + v_s E_{pi} (r_o^2 - r_p^2) q_s - v_p E_{si} (r_o^2 - r_p^2) q_p \text{-----3.16.b}$$

$$p_{oi} = X \cdot p_{1i} + Y \cdot q_s - Z \cdot q_p \text{-----3.17}$$

normalizing eqⁿ 3.17 with respect to q_o ,

$$P_{oi} = X \cdot P_{1i} + Y \cdot Q_s - Z \cdot Q_p \text{-----3.18}$$

where,

$$X = \frac{M_r S^2 (1 + v_s) + M_r (1 - v_s) + (1 - v_p) (S^2 - 1)}{2 \cdot M_r S^2} \text{-----3.19.a}$$

$$Y = \frac{v_s(S^2-1)}{2.S^2} \dots\dots\dots 3.19.b$$

$$Z = \frac{v_p(S^2-1)}{2.M_r S^2} \dots\dots\dots 3.19.c$$

$$M_r = \frac{E_{pi}}{E_{si}} \dots\dots\dots 3.19.d$$

d) Compatibility Requirements :

Compatibility of axial deflection of column and surrounding soil requires that,

$$\delta_s = \delta_p$$

ie,

$$\sum_{i=1}^n \frac{1}{E_{si}} \left[\sigma_z - v_s (\sigma_{\theta i} - \sigma_{\rho i}) \right] \Delta H_i = \sum_{i=1}^n \frac{1}{E_{pi}} \left[\sigma_z - 2v_p \sigma_{\theta i} \right] \Delta H_i \dots\dots\dots 3.20$$

For i^{th} layer,

$$\frac{1}{E_{si}} \left[-q_s - \frac{2v_s}{(r_o^2 - r_p^2)} \left\{ r_p^2 p_{1i} - r_o^2 p_{oi} \right\} \right] = \frac{1}{E_{pi}} \left[-q_p + 2v_p p_{1i} \right] \dots\dots\dots 3.21$$

$$p_{oi} \left\{ 2v_s r_o^2 E_{pi} \right\} = p_{1i} \left\{ 2v_s E_{pi} r_p^2 + 2v_p E_{si} (r_o^2 - r_p^2) \right\} + q_s E_{pi} (r_o^2 - r_p^2) - q_p (r_o^2 - r_p^2) - q_p E_{si} (r_o^2 - r_p^2) \dots\dots\dots 3.22$$

$$p_{oi} = M p_{1i} + N q_s - R q_p \dots\dots\dots 3.23$$

Normalizing it with q_o ,

$$P_{oi} = MP_{1i} + NQ_s - RQ_p \quad \text{-----} 3.24$$

where,

$$M = \frac{2v_s M_r + 2v_p (S^2 - 1)}{2v_s M_r S^2} \quad \text{-----} 3.25.a$$

$$N = \frac{(S^2 - 1)}{2v_s S^2} \quad \text{-----} 3.25.b$$

$$R = \frac{(S^2 - 1)}{2v_s M_r S^2} \quad \text{-----} 3.25.c$$

From the above set of equations Q_p , Q_s , P_{oi} and P_{1i} can be calculated and thus for any value of applied uniform normal stress q_o , the stress q_p , q_s , p_{oi} and p_{1i} can be determined. These stresses have to be determined iteratively and for each layer separately as coefficients X , Y , Z , M , N , and R are different for each layer.

Settlement of foundation on soft clay without stabilizing with stone column can be calculated from constrained modulus approach,

$$(\Delta\delta)_{ut} = q_o \Delta H_i / D_{si} \quad \text{-----} 3.26$$

where,

$$D_{si} = \text{Constrained modulus of soil } (= 1/m_{vi})$$

$$= \frac{(1-v_s)E_{si}}{(1+v_s)(1-2v_s)} \quad \text{-----} 3.27$$

so,

$$(\Delta\delta)_{ut} = \frac{(1+v_s)(1-2v_s)q_o \Delta H_i}{E_{si}(1-v_s)} \quad \text{-----} 3.28$$

The proposed formulation is programmed on a computer and the values of stresses on stone column and soil, q_p and q_s , respectively, and the radial stresses at inner and outer boundaries of unit cell, p_{1i} and p_{oi} , respectively, are calculated. Using these values the axial deformation of each layer is calculated.

The axial deformation of each layer is then added to calculate the net settlement of ground stabilized by stone columns. Settlement of the untreated soil is also calculated and then the settlement reduction factor (S.R.F.) which is the ratio between the difference of settlement of untreated area and treated area to the settlement of untreated area is calculated.

$$\text{S.R.F.} = \frac{\delta_{ut} - \delta_t}{\delta_{ut}} \times 100\% \quad \text{.....(28)}$$

3.2 PART 2 :

SHEAR STRENGTH ANALYSIS OF UNIT CELL

General stability of the earth mass is often a serious problem when embankments are constructed over soft underlying soil. An important use of stone column is to improve the marginal sites to permit the construction of embankments and also to stabilize the existing slopes, undergoing landslide problems. A stability analysis of an embankment using stone columns can be performed in the same manner as for a normal slope stability problem, except stress concentration in the stone column must be considered.

3.21 METHOD OF ANALYSIS: -

The average shear strength method is one of the techniques that can be used to analyze the stability of stone column reinforced ground and is used here in our analysis. The weighted average material properties are calculated for the material within the unit cell. The soil having the fictitious weighted material properties is then used in the analysis.

Fig.3.4 shows the embankment, founded on stone column reinforced ground, with the sliding surface. Stone column material is considered to have only internal friction ϕ_p and no cohesion, whereas the surrounding soil is undrained but has both cohesion C_u and angle of internal friction ϕ_{su} . The forces acting on a typical slice of stone column are shown in Fig.3.5 .

The vertical stress due to self weight and due to the applied uniform load is,

$$\sigma_{pz} = q_p + \gamma_p \cdot Z \quad ; \text{for stone column} \quad \text{-----} 3.30.a$$

$$\sigma_{sz} = q_s + \gamma_s \cdot Z \quad ; \text{for surrounding soil} \quad \text{-----} 3.30.b$$

where,

σ_{pz} = Vertical stress on sliding surface of a stone column.

σ_{sz} = Vertical stress on sliding surface of surrounding soil.

γ_p and γ_s = The unit weight of column material and soil respectively.

Z = Depth below the ground surface.

The average shear strength of the unit cell is given by ,

$$\tau_{av} = T/a_o \quad \text{-----} 3.31$$

$$\text{and,} \quad T = a_p \tau_p + (a_o - a_p) \tau_s \quad \text{-----} 3.32$$

where,

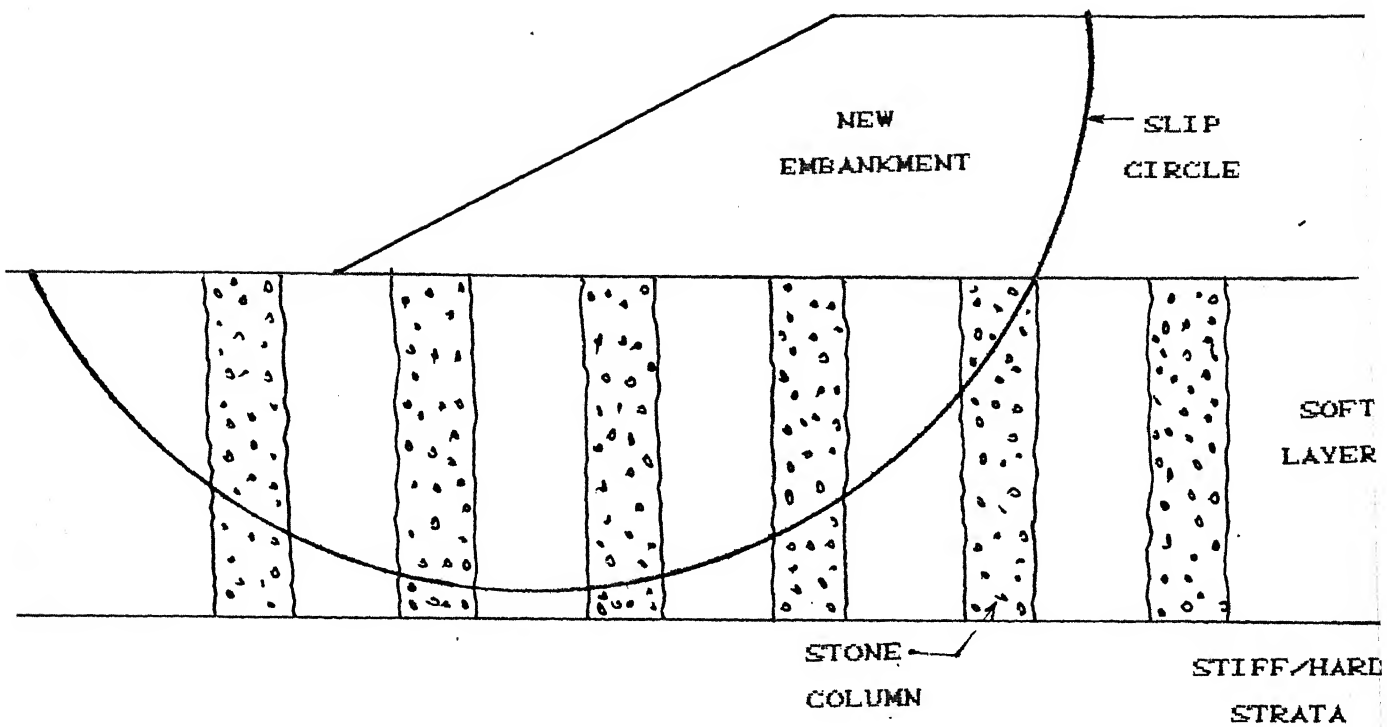


Fig.3.4 EMBANKMENT FOUNDED ON STONE COLUMN REINFORCED GROUND

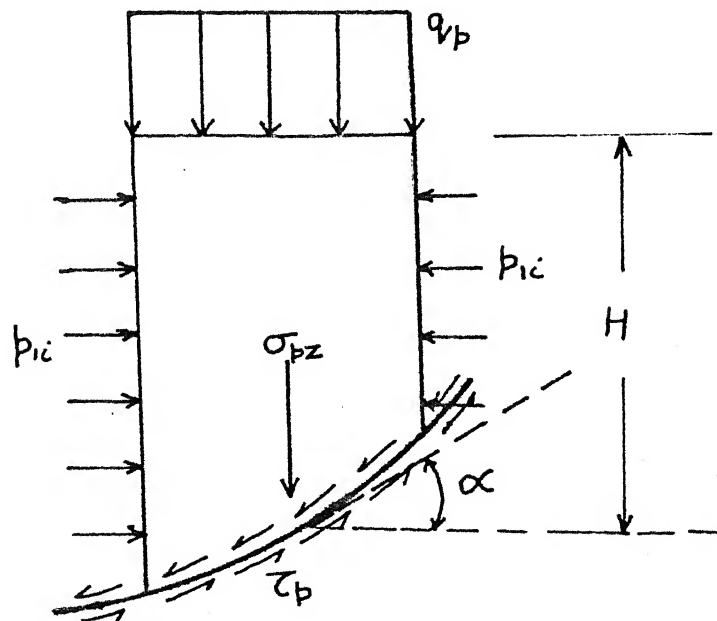


Fig.3.5 FORCES ACTING ON A TYPICAL SLICE OF STONE COLUMN

T = Average shear force

a_o = Cross sectional area of unit cell ($= \pi d_o^2/4$)

a_p = Cross sectional area of stone column ($= \pi d_p^2/4$)

τ_p = Shear strength of stone column

τ_s = Shear strength of surrounding soil

therefore,

$$\tau_{av} = (a_p/a_o) \cdot \tau_p + (1-a_p/a_o) \cdot \tau_s \quad \text{-----} 3.33$$

where, a_p/a_o is the area ratio $= 1/S^2$

$$\tau_{av} = (1/S^2) \cdot \tau_p + (1-1/S^2) \cdot \tau_s \quad \text{-----} 3.34$$

Since the failure surface is inclined at an angle, α , therefore inclined cross sectional area is to be considered. Fortunately the area ratio remains $1/S^2$.

The shear strength of stone column is given by,

$$\tau_p = [\sigma_{pz} \cdot \cos^2 \alpha + p_{1i} \sin^2 \alpha] \tan \phi_p \quad \text{-----} 3.35$$

$$= [(q_p + \gamma_p z) \cdot \cos^2 \alpha + p_{1i} \cdot \sin^2 \alpha] \tan \phi_p \quad \text{-----} 3.36$$

The shear strength of surrounding soil is given by ,

$$\tau_s = C_u + (q_s + \gamma_s z) \cdot \cos^2 \alpha \cdot \tan \phi_{su} \quad \text{-----} 3.37$$

Therefore the average shear strength ,

$$\tau_{av} = (1/S^2) \cdot \left[(q_p + \gamma_p Z) \cdot \cos^2 \alpha + p_{1i} \cdot \sin^2 \alpha \right] \tan \phi_p + (1 - 1/S^2) \cdot \left[C_u + (q_s + \gamma_s Z) \cdot \cos^2 \alpha \cdot \tan \phi_{su} \right] \quad \text{-----} \quad 3.38$$

Representing it in the following form,

$$\tau_{av} = C_e + q_o \cdot \tan \phi_e \quad \text{-----} \quad 3.39$$

where,

C_e = Equivalent cohesion of reinforced soil

$$= (1 - 1/S^2) \cdot C_u \quad \text{-----} \quad 3.40.a$$

ϕ_e = Equivalent angle of internal friction of reinforced soil.

$$= \tan^{-1} \left[(1 - 1/S^2) \left\{ \left\{ \frac{q_s + \gamma_s \cdot Z}{q_o} \right\} \cdot \cos^2 \alpha \cdot \tan \phi_{su} \right\} + (1/S^2) \cdot \right.$$

$$\left. \left\{ \left\{ \frac{q_p + \gamma_p \cdot Z}{q_o} \right\} \cdot \cos^2 \alpha + p_{1i}/q_o \cdot \sin^2 \alpha \right\} \cdot \tan \phi_p \right]$$

----- 3.40.b

CHAPTER IV

RESULTS AND DISCUSSIONS

In this chapter the results obtained from the theoretical analysis are presented and discussed. The details of the model studies carried out in the laboratory are presented. The experimental results are then compared with the theoretical results. Results obtained from this analysis show that the most important parameters affecting the settlement behaviour of stone column reinforced soil are, the vertical stress (q_o), the column-soil spacing ($S = d_o/d_p$), the modular ratio of stone column and soil ($M_r = E_p/E_s$) and the poisson's ratio of soil (v_s). The effect of these parameters are shown in the form of graphs. The load-settlement behaviour of stone column reinforced soil and untreated soil is presented and compared in terms of settlement reduction factor.

From the shear strength analysis of a unit cell, design charts are obtained for the variation of the ratio of equivalent shear strength of treated soil to the undrained shear strength of treated soil with the applied normal stress. These design charts are obtained for various values of S , M_r , and α , the inclination of the failure surface.

The analysis is carried out for, $M_r = 5, 10, 25$ and 50 ; $S = 2.0, 2.5$ and 3.0 ; the rate of increase of elastic modulus of soil with the normal stress, $A_s = 5, 15$, and 25 . The poisson's ratio of the soil and that of the stone column is taken

as 0.4 and 0.3 respectively. The length of stone column for settlement analysis is taken 5.0 meter and the unit cell is divided into ten layers. The soil is considered to follow a linear stress-strain law but its modulus of deformation increases with the depth ie, $E_s = E_s(z)$. In the analysis it has been considered that at 5.0 meter depth the elastic modulus becomes 2.5 times than near the surface.

4.1 RESULTS OF THEORETICAL ANALYSIS :

The effect of spacing and the modular ratio of stone column and soil on the contact stresses of stone column (q_p/q_o), is shown in Fig 4.1, 4.2 and 4.3. These plots are drawn for $A_s = 5, 15$ and 25 . The stress on stone column increases rapidly with the modular ratio. This is because of the increased rigidity of the stone column and therefore more load is carried by the stiffer material. As the diameter of the stone column decreases the stress on stone column increases because of the lesser area of stone column. From these sets of curves it is also clear that as the value of A_s increase, stress on stone column decreases. This is because the deformation modulus of soil increases with, A_s , at any stress level and therefore the soil becomes stiffer and carries more load.

Fig 4.4 and Fig 4.5 depict the variation of stress on stone column and the soil with the normal stress respectively for different values of modular ratio. These results suggest that the stone column can be used most effectively when the rigidity of the stone column is high as compared to the surrounding soil.

The settlement behaviour of stone column

treated area and settlement reduction factor which is the ratio between the difference of settlement of untreated and treated soil to the settlement of untreated soil with the normal stress are shown in Fig 4.6 to Fig 4.14 . The effect of modular ratio of stone column and soil, and the spacing of stone column is shown. These plots are drawn for different values of S and A_s . From these graph it can be observed that by increasing stone column diameter or by decreasing S , the settlement of treated soil decreases significantly. Further for the higher value of modular ratio, there will be a rapid increase in the settlement reduction factor. In other words the settlement of the treated soil is very small as compared to the untreated soil. This is because of the fact that the stress on soil becomes less as the stone column of higher deformation modulus is used . For example when $S = 2.0$, $A_s = 15.0$ and $q_0 = 100$ kN/sqm then the settlement reduction factor for $M_r = 10$ is 23%, for $M_r = 25$ it is 56% and for $M_r = 50$ it becomes 76% . For increasing values of normal stress, q_0 , the S.R.F. also increases. For smaller values of modular ratio it increases rapidly than for higher values of modular ratio. This is because the ratio of stress on stone column and soil varies within a wide range for lower modular ratio . For example, consider the case in which $S = 2.0$, $A_s = 15.0$ and normal stress is ranging from 10 kN/sqm to 200 kN/sqm . For $M_r = 10$, the ratio of stresses on stone column and soil (q_p/q_s and q_s/q_0 respectively) vary from 2.388 - 1.454 and 0.537 - 0.849 respectively, whereas for $M_r = 50$, this range is 3.496 - 3.21 and 0.157 - 0.263 only .

The results from one dimensional model of stone column - soil unit without considering the radial stresses

are shown in Fig 4.15 . The ratio of stone column diameter to the diameter of the unit cell is plotted against the settlement of treated ground divided by the settlement of untreated ground . This graph is for $\nu_p = \nu_s = 0.3$. From this graph it is clear that when there is no stone column or $d_p/d_o = 0$, then there is no reduction in settlement or $\delta_t/\delta_{ut} = 1$ and when $d_p/d_o = 1$, ie, when the soil is completely replaced by stone column, the settlement reduction is very high . It is more for higher modular ratio . For $M_r = 50$, a settlement reduction upto 96% may be expected .

Fig 4.16 shows the variation in the magnitude of stress on stone column and surrounding soil from the undrained to drained condition . When the load is initially applied to the stabilized soil deposit the soil will deform under undrained condition as an incompressible material (poisson's ratio $\nu_s = 0.5$) whereas the high permeability of the stone column material ensures that it deform under drained condition . Therefore initially when the load is applied the stress on the soil may be greater than on the stone column . But consolidation of soil by radial flow towards the stone column results in the soil deforming under drained condition as a compressible material ($\nu_s < 0.5$). Therefore the soil becomes less stiffer than the granular material of stone column and stress on stone column is greater than on soil .

Fig 4.17, 4.18 and 4.19 illustrates the effect of the angle of inclination of sliding surface (α), modular ratio of stone column and soil (M_r), column spacing (S) and the normal stress (q_o) on the ratio of equivalent shear strength of stone column - soil unit to the undrained shear strength of soil . These three graphs are drawn for $S = 2, 2.5$ and 3

respectively . When the angle of inclination of sliding surface is very small then modular ratio has a very significant effect on equivalent shear strength , whereas at very high angles of inclination of the sliding surface the effect of modular ratio on equivalent shear strength is not significant . From these plots it may also be observed that by increasing the stone column diameter, the equivalent shear strength increases which is reasonable .

4.2 MODEL STUDIES :-

The direct shear apparatus is used to carryout the model studies in the laboratory . The group effect of the stone column can best be considered through a single stone column - soil unit . This unit cell is approximated by direct shear box of size 6 cm x 6 cm x 2.5 cm . This square size of the sampler can be approximated to an equivalent area of circular sampler which can fulfill the condition of cylindrical stone column - soil unit of diameter $d_o = 6.77$ cm.

4.2.1 SOIL AND GRANULAR MATERIAL PROPERTIES :-

Local Kanpur silt and Kalpi sand were used in the studies . Various laboratories tests were conducted to determine the index properties of soils . To determine the strength parameters of the local silt and sand which are used in the model tests, consolidated undrained direct shear tests were conducted . For the determination of coefficient of consolidation of silt and sand , oedometer tests were conducted in the direct shear box itself . The properties of local silt and sand are as follows :-

Local Silt :

Liquid limit (w_l) = 29.0

Plastic limit (w_p) = 16.6

Plasticity index P.I. = 12.4

Specific Gravity (G) = 2.69

Unit Weight (γ_s) = 18 kN/sqm

Cohesion (C_{cu}) = 20 kN/sqm

Angle of Internal Friction (ϕ_{cu}) = 12°

Coefficient of consolidation (C_{cc}) = 0.29

Initial Void Ratio (e_{oc}) = 0.92

TABLE 4.1

Granular Material

PERCENTAGE FINER

Sieve Size (mm)	1.0	0.6	0.425	0.16	0.015
Sand	100.0	93.4	68.2	13.7	2.1

Cohesion $C_p = 0$

Angle of Internal Friction $\phi_p = 35^\circ$

Unit Weight $\gamma_p = 19$ kN/sqm

Coefficient of Consolidation (C_{cp}) = 0.05

Initial Void Ratio (e_{op}) = 0.49

4.2.2 TESTING DETAILS :-

The local silt sample taken in the sampler and with the help of a very thin hollow cylindrical steel tube, a hole of required diameter is made at the center of the sample. This

cylindrical hole is then filled with the granular material (sand) in layers and compacted such that it is having the same density as that of sand which was tested alone in the direct shear apparatus. This soil-column unit is then placed in the direct shear apparatus and filled with water to saturate and consolidate under a nominal pressure of 0.05 kg/sq cm for 24 hours. This unit is then consolidated under a normal stress and sheared without permitting drainage. The test values of normal pressures for which this column-soil unit is sheared are 25, 50, 75 and 100 kN/sqm. The sheared tests were carried out for the various values of column diameter, d_p , such that the ratio between the diameter of the unit cell to the diameter of column, S , becomes 2.0, 2.5 and 3.0. For this the columns were made of the diameter 3.39, 2.71 and 2.25 cms respectively.

The stress-strain behaviour of silt and sand are shown in Fig. 4.20 and 4.21 respectively. The value of M_r , the ratio between the elastic modulus of granular material to the elastic modulus of soil, is in between 5-10 for the test range.

For different values of S and normal stress, q_0 , the load shearing between the column and the soil can be determined. The test results obtained from the experimental analysis and theoretical analysis are compared and discussed in the following section.

4.3 RESULTS OF THE EXPERIMENTAL ANALYSIS AND COMPARISON WITH THE THEORETICAL RESULTS:-

The stress-strain behaviour of the soil and the sand used in the model studies are shown in the Fig. 4.20 and

Fig. 4.21 respectively. The test is performed for confined compression condition where no radial movements are allowed during the loading. From these curves it has been observed that the ratio of elastic modulus of sand to soil is in between 5-10 for the test range of normal stress upto 200 kN/sqm..

From the failure envelope, cohesion and angle of internal friction for soil, sand and soil-sand unit are determined. For different values of S , the equivalent angle of internal friction (ϕ_e) of combined mass is shown in the Fig. 4.29.

From model studies, the stress distribution between stone column and soil for different values of column spacing, S , are illustrated in Fig. 4.22 and Fig. 4.23. These two plots are drawn for normal stress of 25 kN/sqm and 75 kN/sqm respectively. Analytical results are also drawn to compare the experimental results. The experimental values justify the theoretical results as they lie in between the theoretical results for modular ratio of 5 and 10. Further, Fig. 4.24 and Fig. 4.25 depicts the variation of stress on stone column and surrounding soil, q_p/q_o and q_s/q_o respectively, with the normal stress q_o . These experimental plots may be compared with the theoretical plots (Fig. 4.4 and Fig. 4.5)

Fig. 4.26 to Fig.4.28 show the plot between normal stress and maximum shear stress for the unit cell, stone column and the soil. These three plots are drawn for $S = 2, 2.5$ and 3 . The direct shear test results on unit cell are compared with the theoretical results. The test points lie in between the theoretical strength envelopes corresponding to the modular ratio of 5 and 10, which verifies the theoretical results. The strength

envelopes for soil and sand are drawn, from where it can be observed that upto the stress level of 40 kN/sqm, the strength of stone column alone is less than that of the soil alone. From these plots it is interesting to note that for a particular range of normal stress, shear strength of the combined stone column-soil mass is greater than both the stone column and soil separately. The reason of this behaviour of the unit cell may be due to the fact that for this range of normal stress level the shear on stone column is very high due to lesser area of stone column. This stress on stone column corresponds to greater shear strength of gravel material alone. Therefore the average shear strength might be greater than both the stone column and soil alone.

The variation of the equivalent angle of internal friction (ϕ_e) with column spacing, S , is shown in Fig. 4.29. From this plot it may be observed that when S becomes 1 or soil is completely replaced by stone column then the angle of internal friction will be 35° whereas for the large values of S it becomes nearly 12° which is the value of angle of internal friction of soil only.

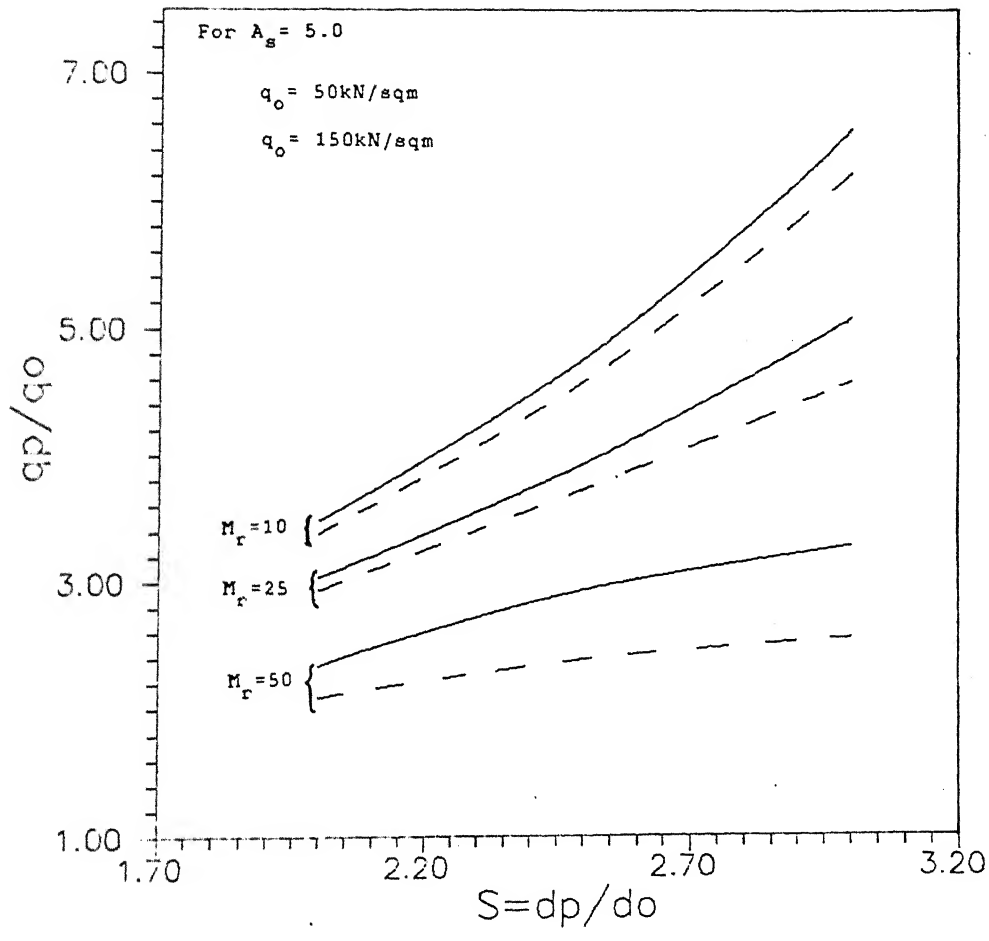


Fig. 4.1 VARIATION OF STRESSES ON STONE COLUMN WITH COLUMN SPACING.

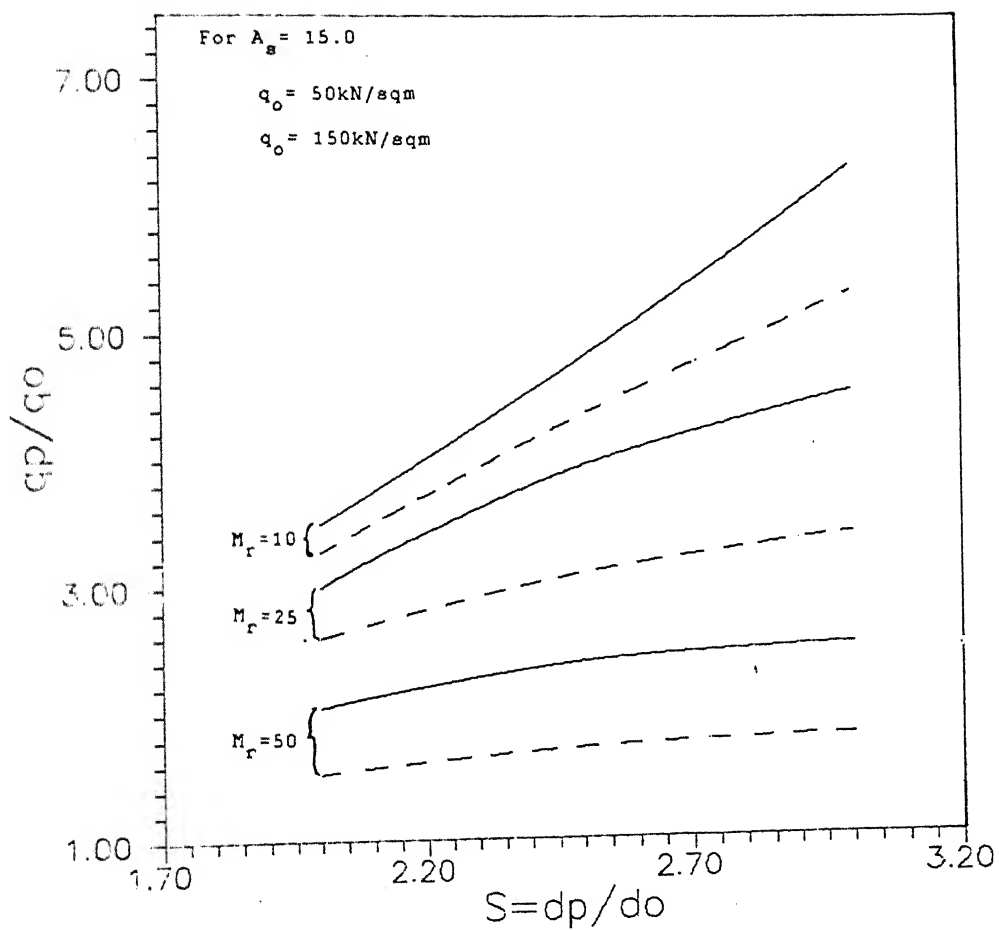


Fig.4.2 VARIATION OF STRESSES ON STONE COLUMN WITH COLUMN SPACING.

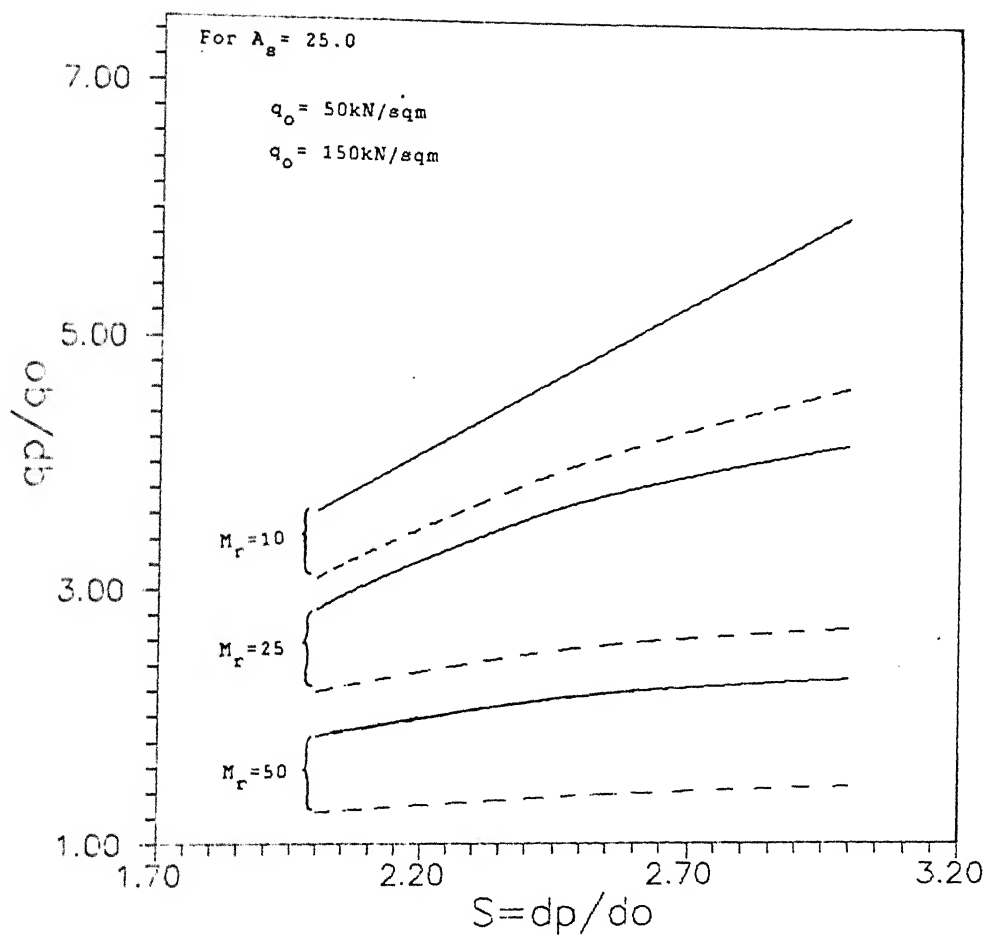


Fig.4.3 VARIATION OF STRESSES ON STONE COLUMN WITH COLUMN SPACING.

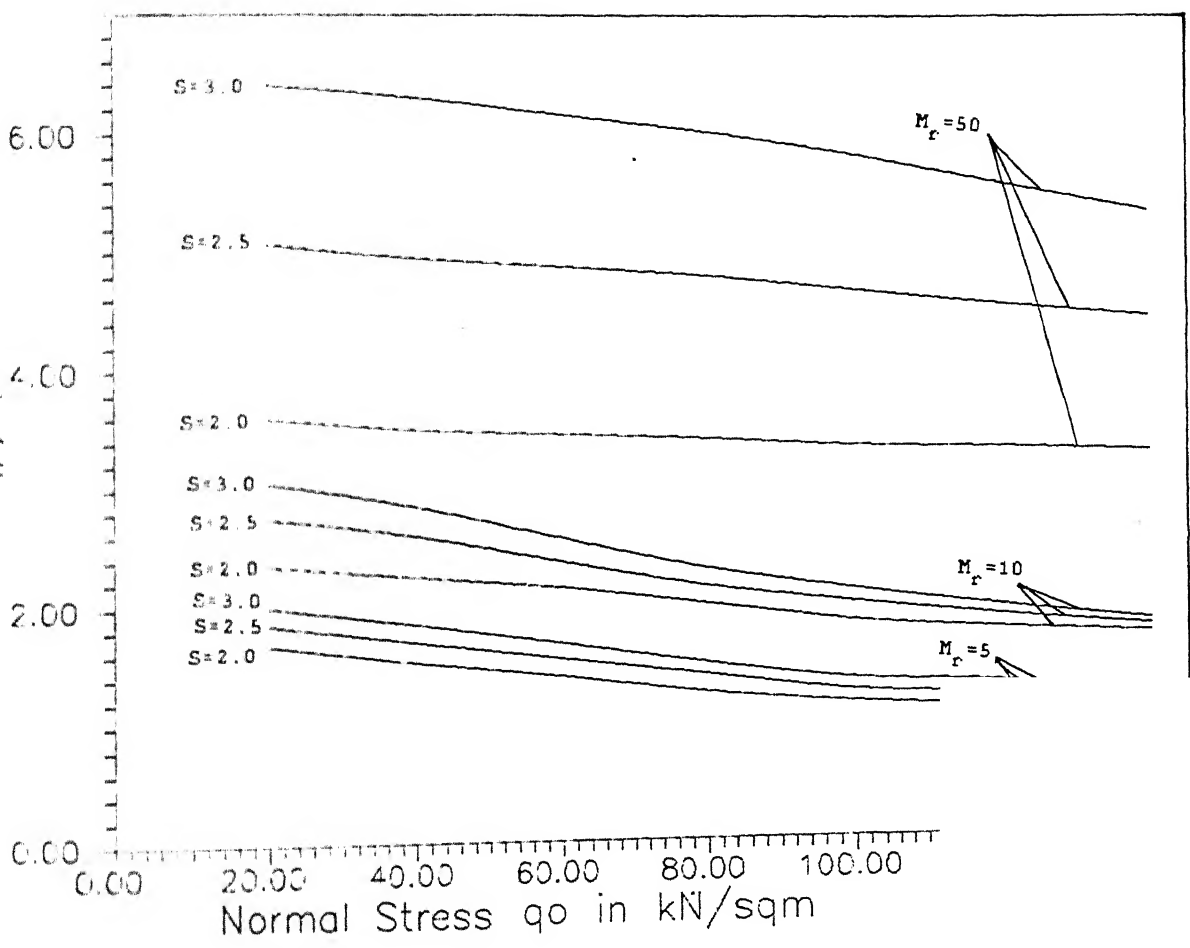


Fig.4.4 VARIATION OF STRESSES ON STONE COLU
THE NORMAL STRESS.

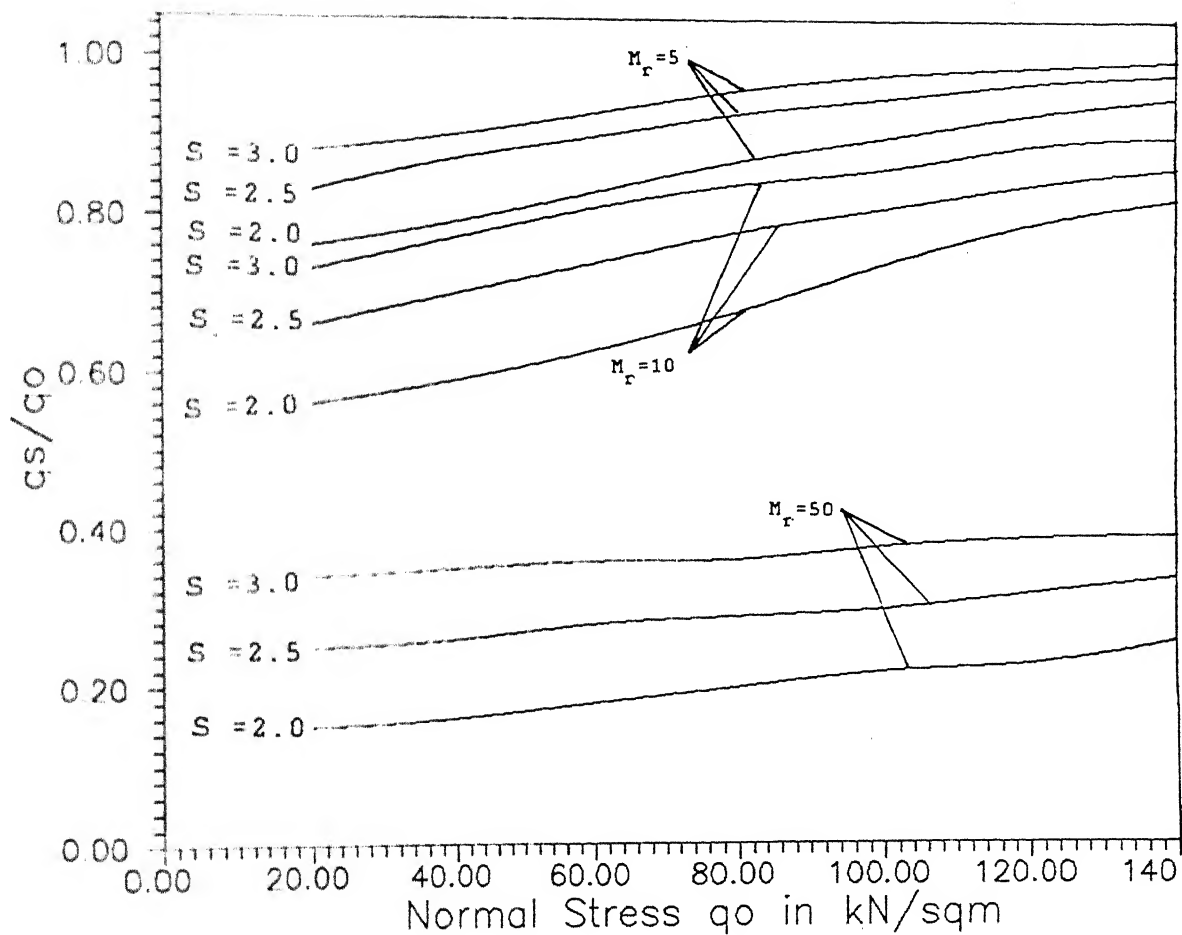


Fig.4.5 VARIATION OF STRESSES ON SOIL WITH THE NORMAL STRESS.

$S = 2.0$
 $A_g = 5$

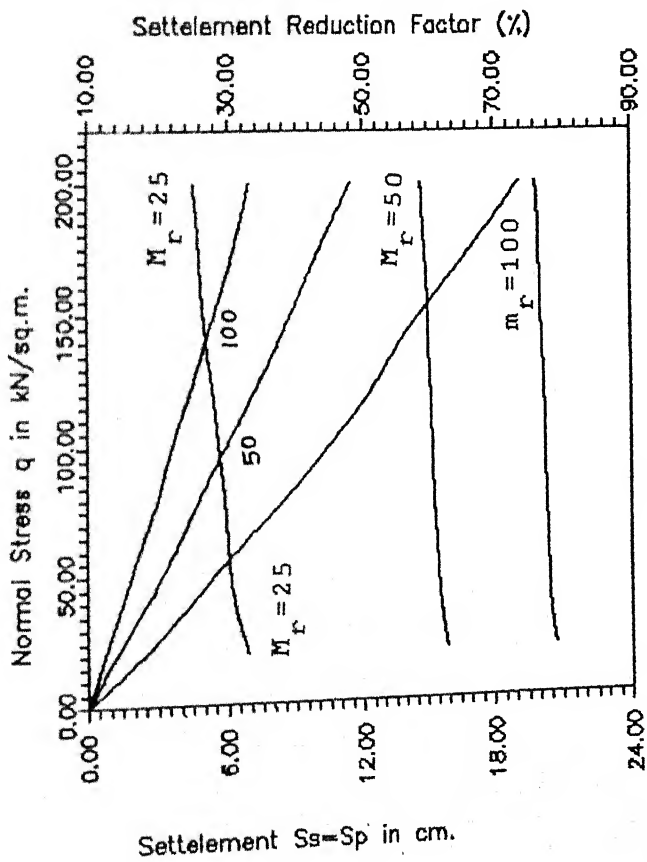


Fig. 4.6

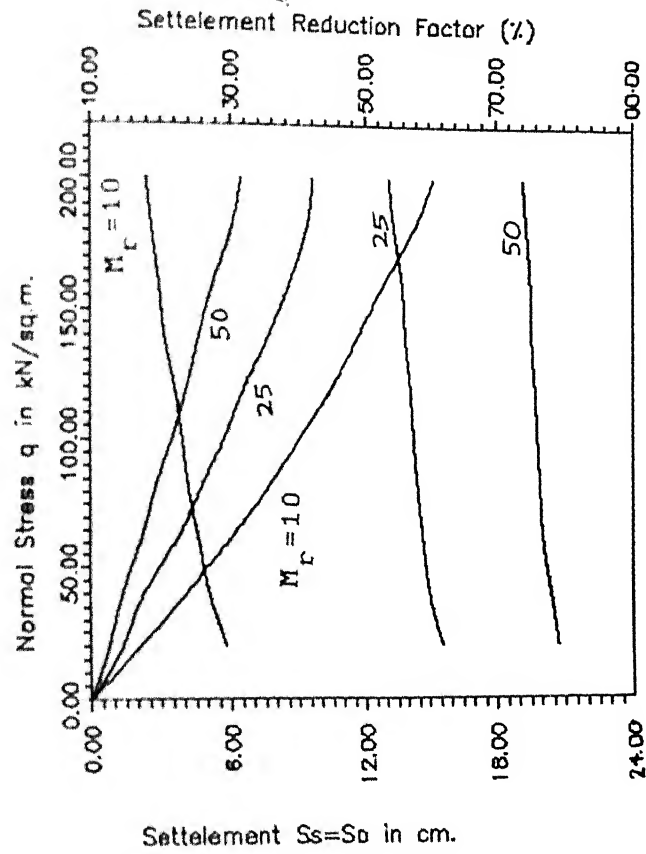


Fig. 4.7

SETTLEMENT BEHAVIOUR OF STONE COLUMN TREATED AREA AND SETTLEMENT
 REDUCTION FACTOR.

$S = 2.5$
 $A_g = 5$

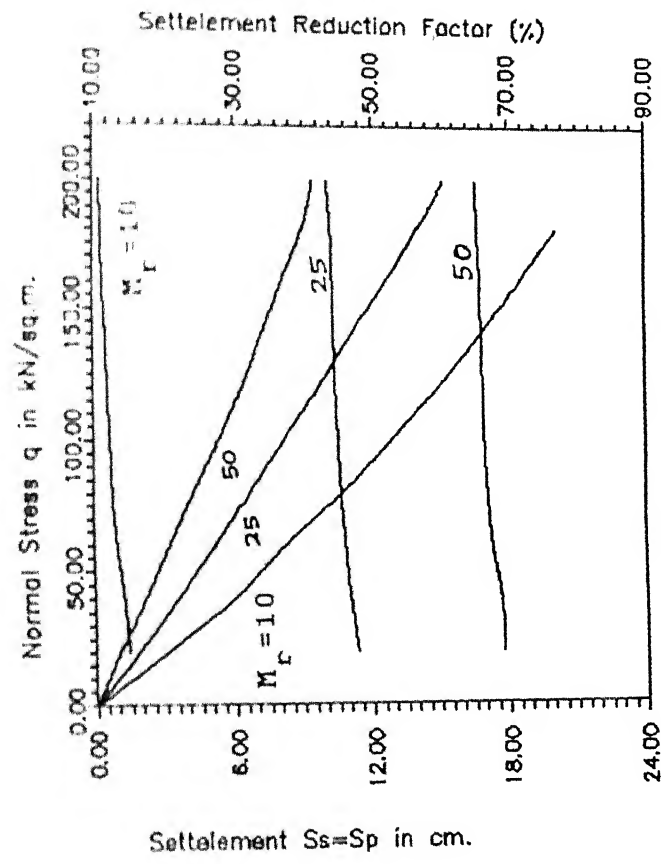


Fig. 4.9

$S = 2.0$
 $A_g = 25$

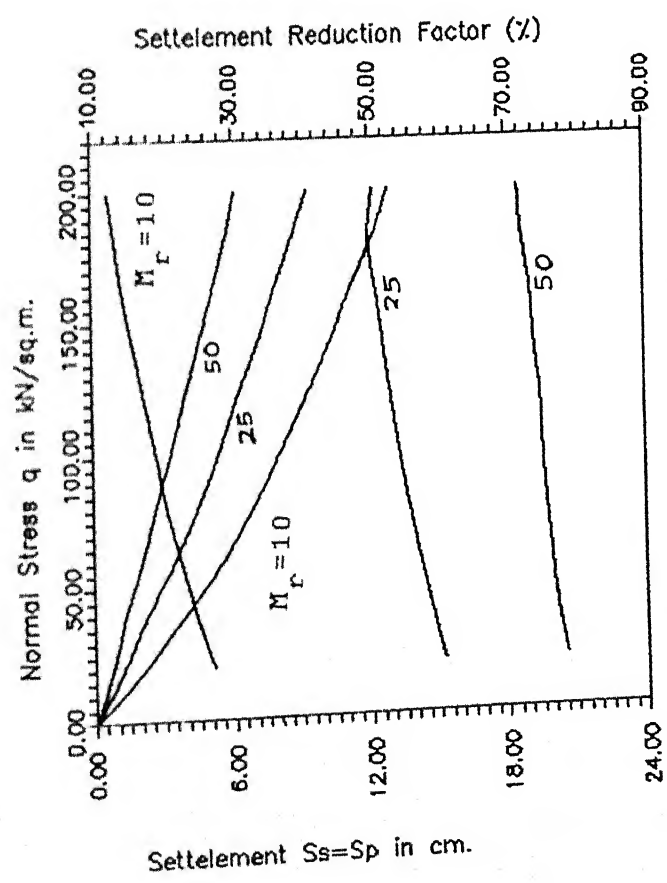


Fig. 4.8

SETTLEMENT BEHAVIOUR OF STONE COLUMN TREATED AREA AND SETTLEMENT
 REDUCTION FACTOR.

SETTLEMENT BEHAVIOUR OF STONE COLUMN TREATED AREA AND SETTLEMENT REDUCTION FACTOR.

$S = 2.5$
 $A_s = 25$

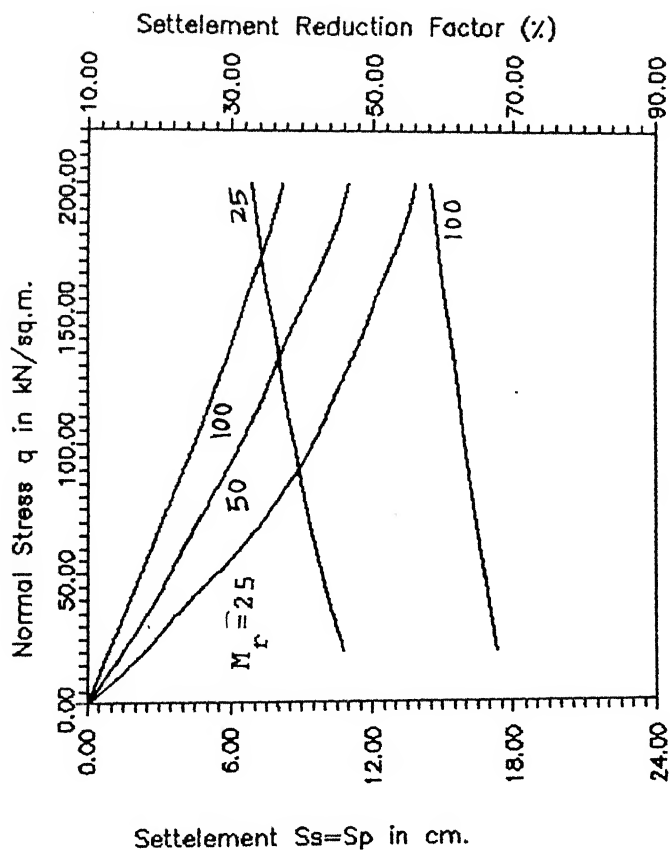


Fig. 4.11

$S = 2.5$
 $A_s = 15$

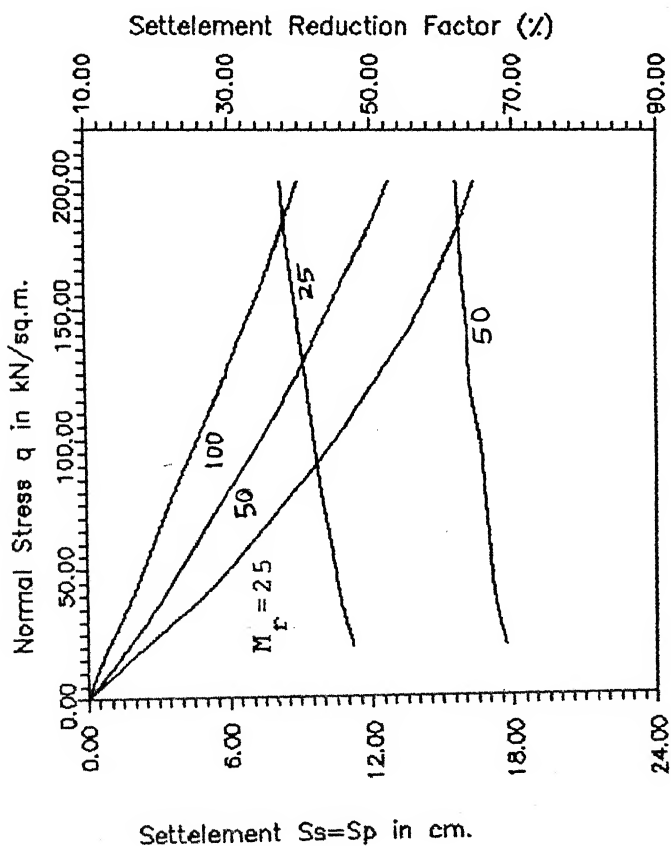


Fig. 4.10

$S = 3.0$

$A_g = 15$

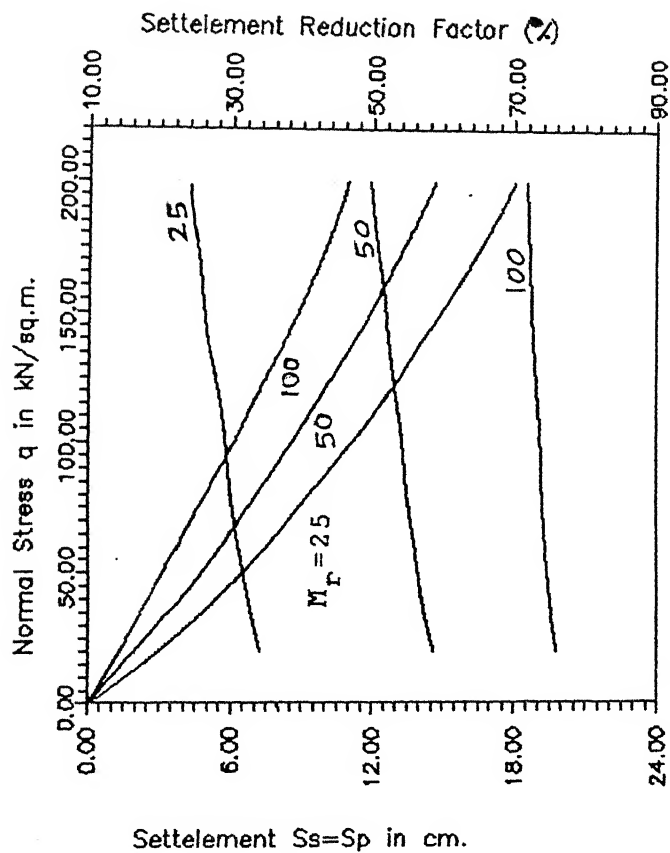


Fig. 4.13

$S = 3.0$

$A_g = 5$

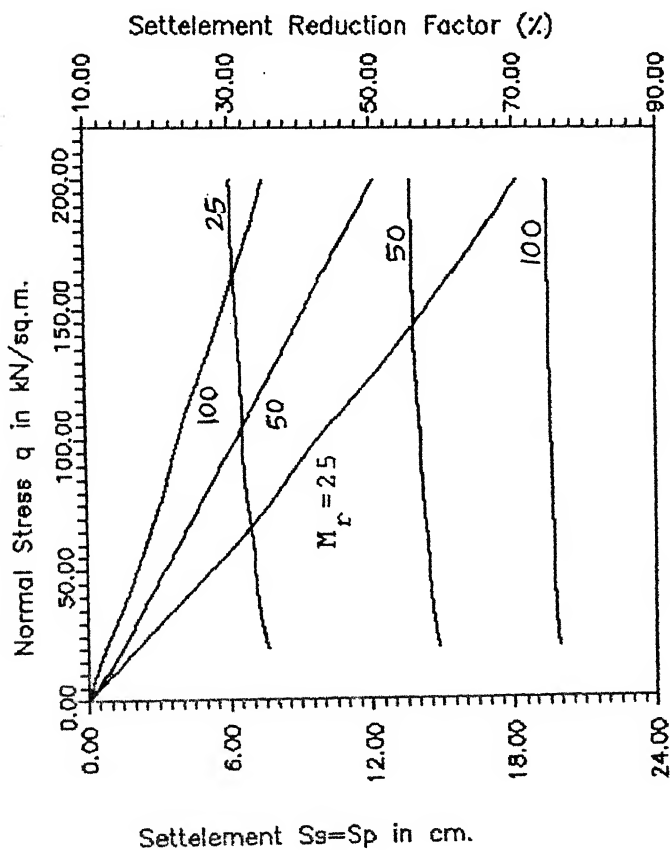


Fig. 4.12

SETTLEMENT BEHAVIOUR OF STONE COLUMN TREATED AREA AND SETTLEMENT
REDUCTION FACTOR.

$S = 3.0$

$A_g = 25$

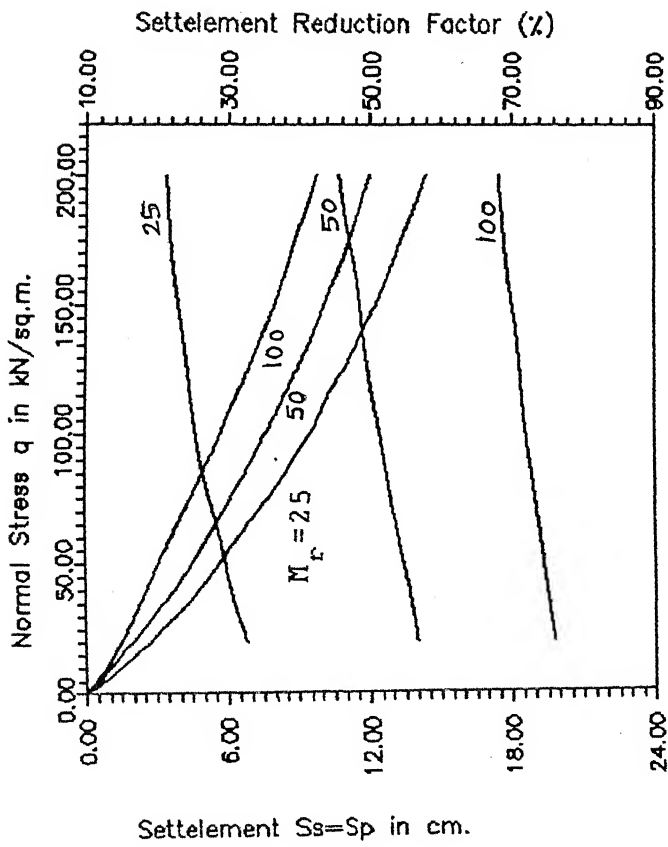


Fig. 4.14

SETTLEMENT BEHAVIOUR OF STONE COLUMN TREATED AREA AND SETTLEMENT
REDUCTION FACTOR.

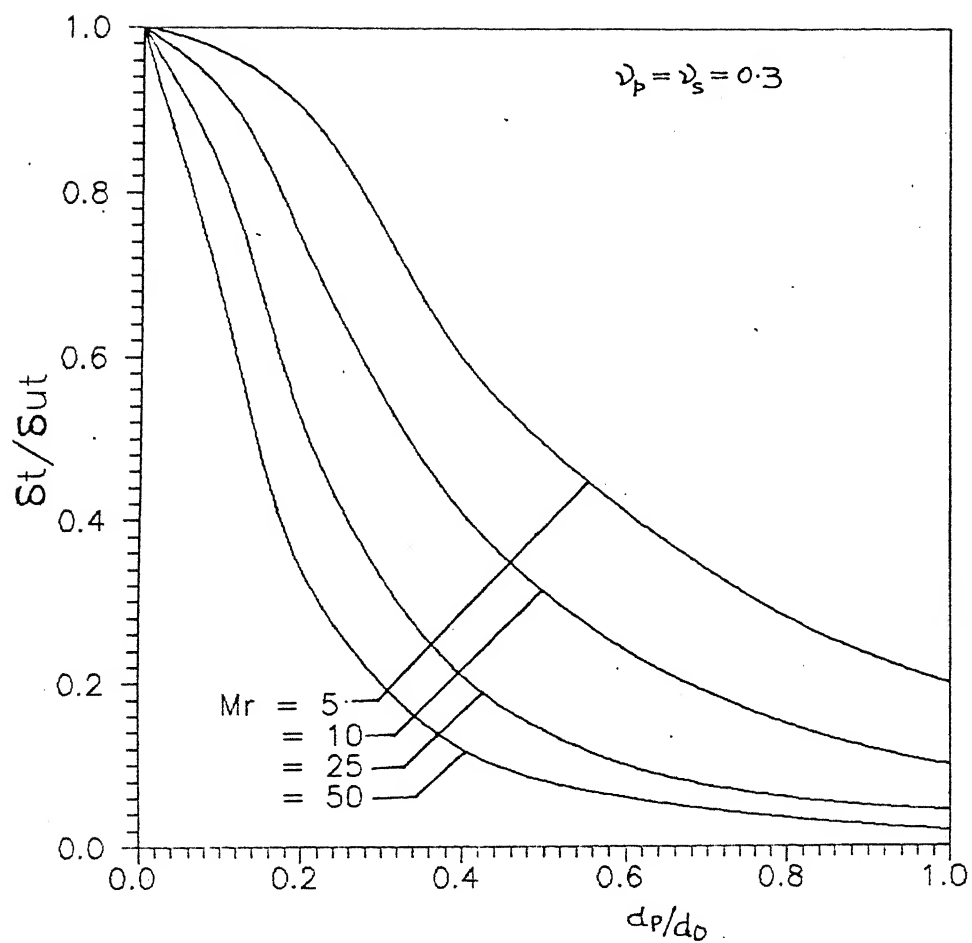


Fig.4.15 EFFECT OF SPACING RATIO (d_p/d_o) ON THE SETTLEMENT REDUCTION (δ_t/δ_{ut}).

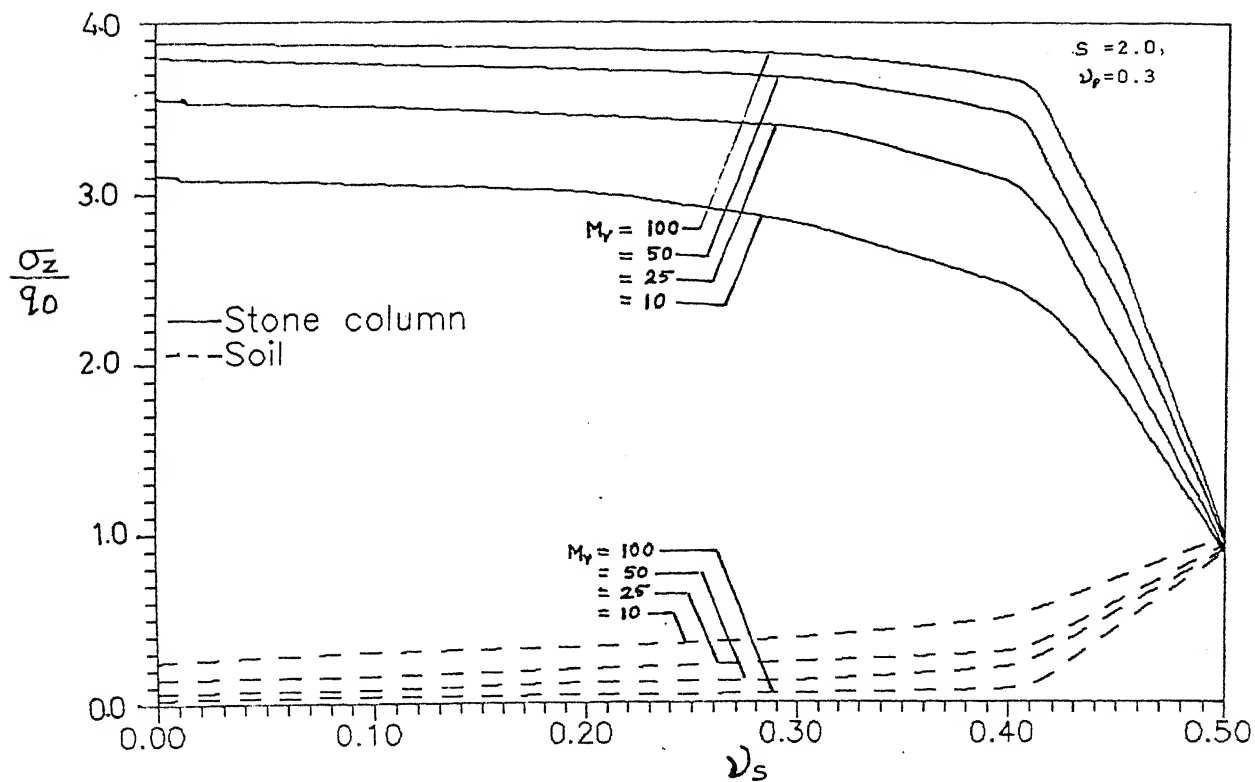


Fig.4.16 VARIATION OF NORMAL STRESSES ON STONE COLUMN
AND SOIL WITH POISSON'S RATIO OF SOIL.

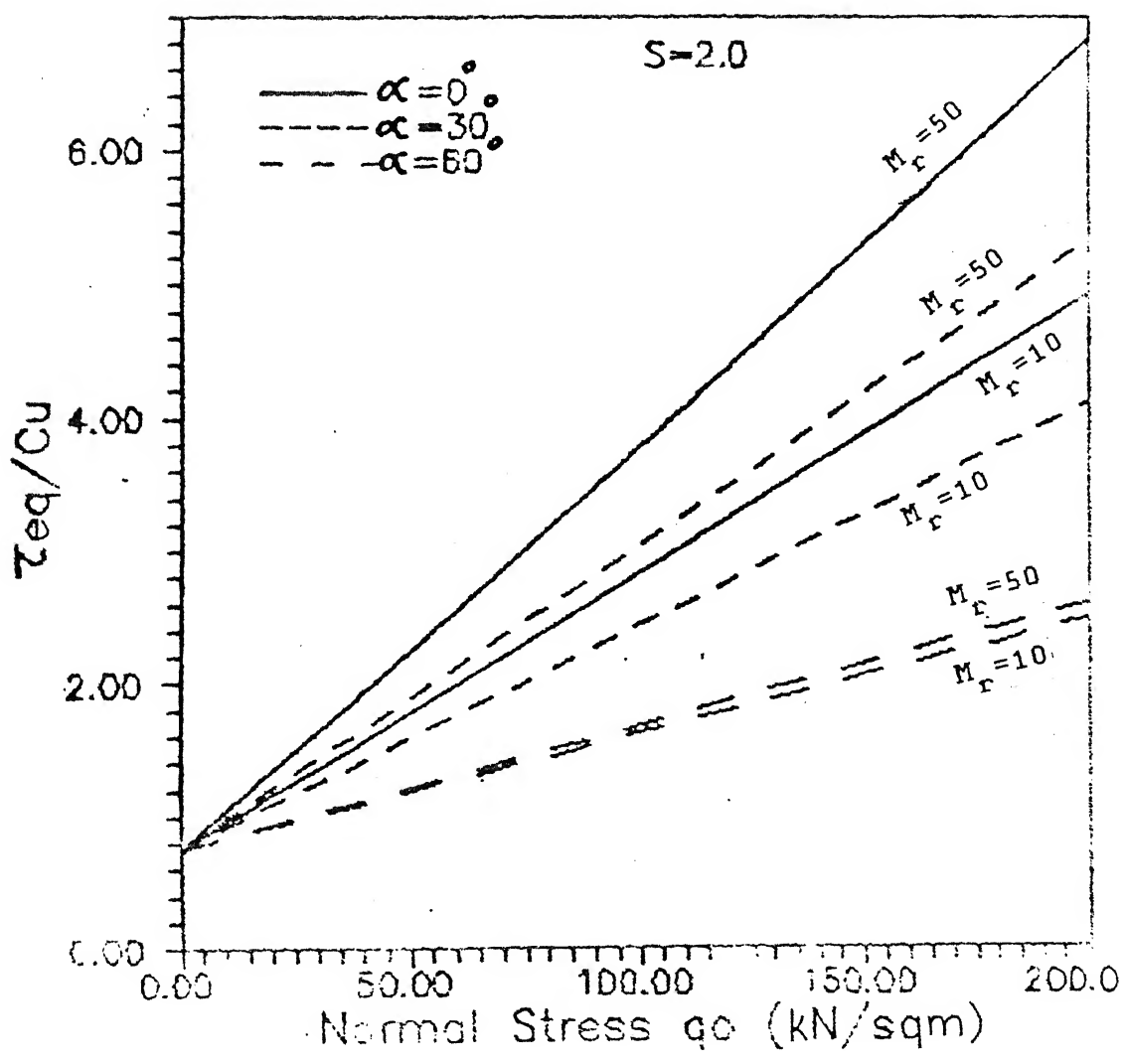


Fig.4.17 EFFECT OF MODULAR RATIO AND INCLINATION OF SLIDING SURFACE ON THE RATIO τ_{eq}/C_u .

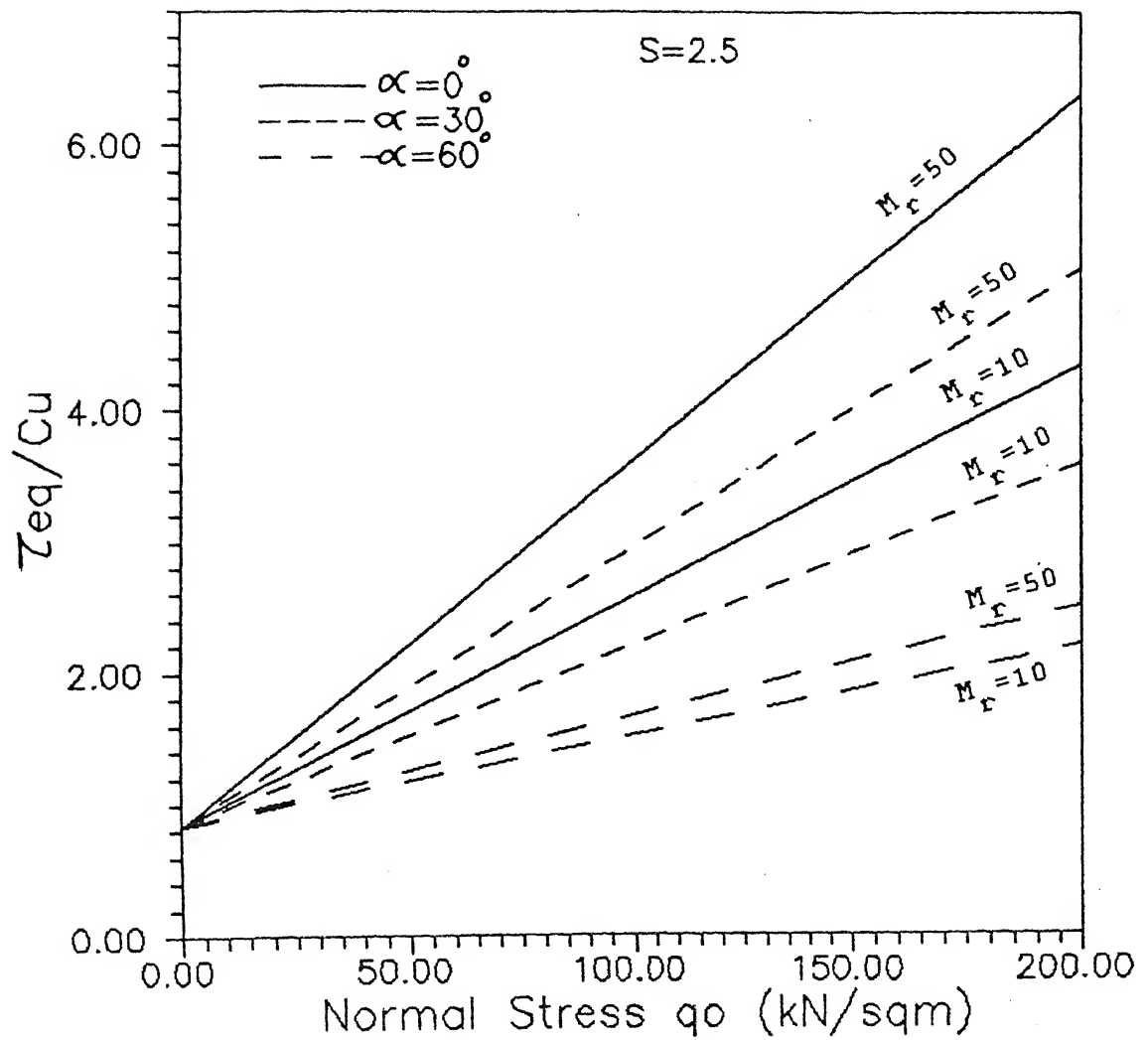


Fig.4.18 EFFECT OF MODULAR RATIO AND INCLINATION OF SLIDING SURFACE ON THE RATIO τ_{eq}/C_u .

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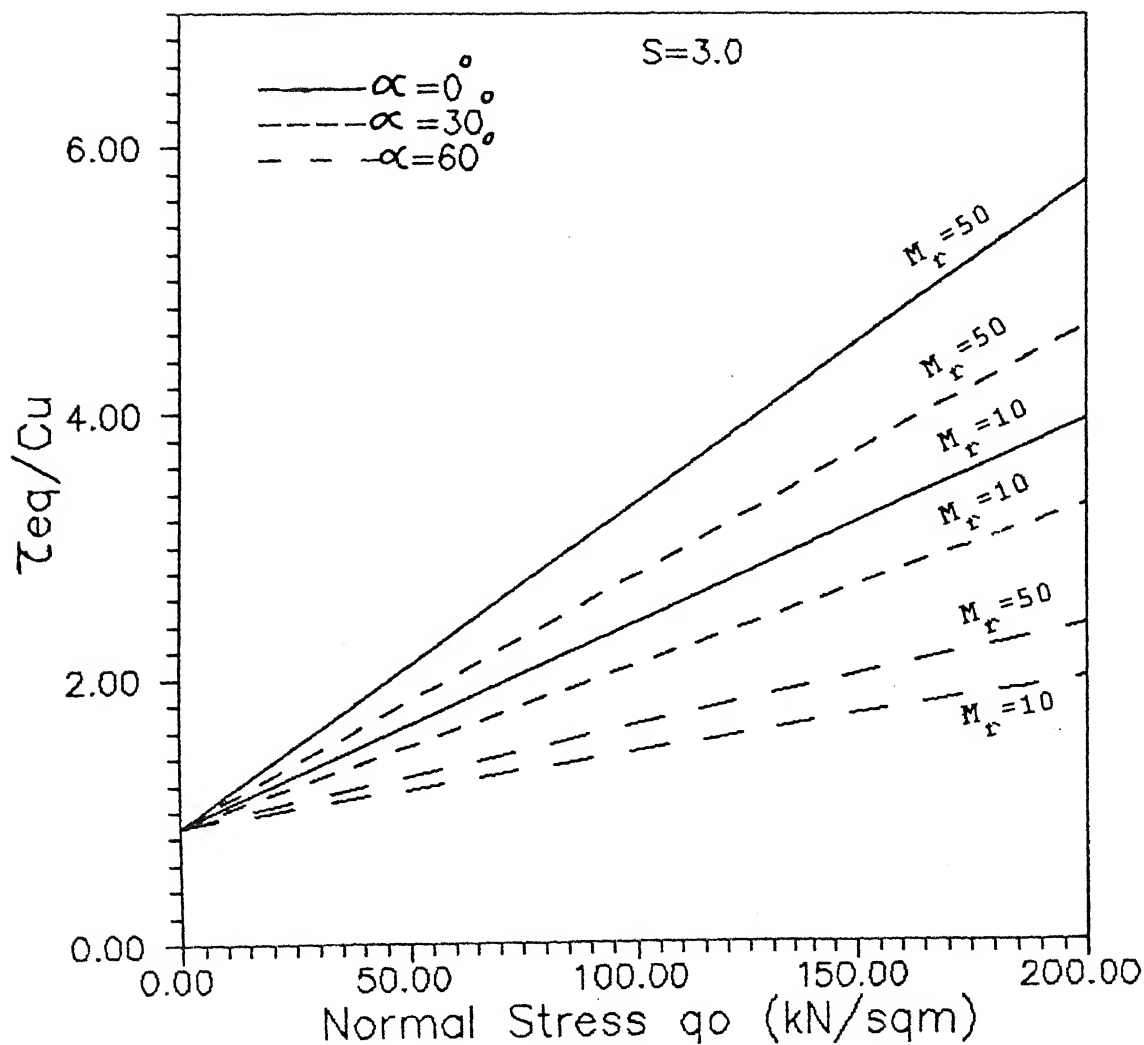


Fig.4.19 EFFECT OF MODULAR RATIO AND INCLINATION OF SLIDING SURFACE ON THE RATIO τ_{eq}/C_u .

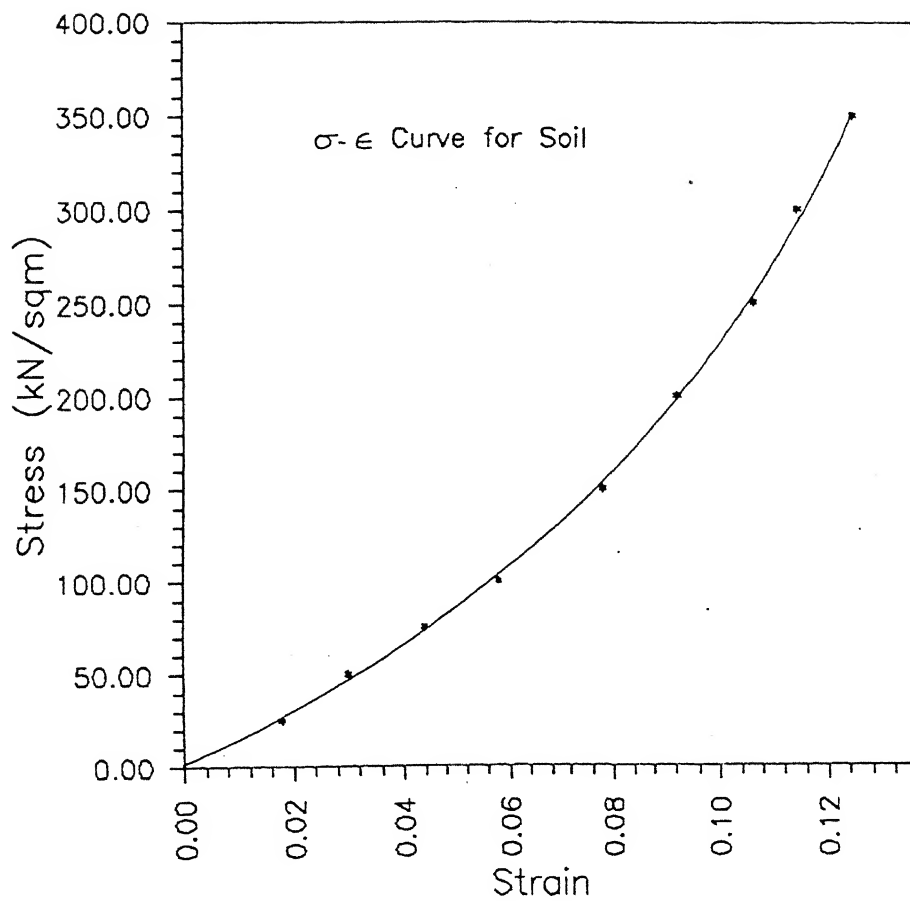


Fig.4.20 STRESS-STRAIN BEHAVIOUR OF SOIL

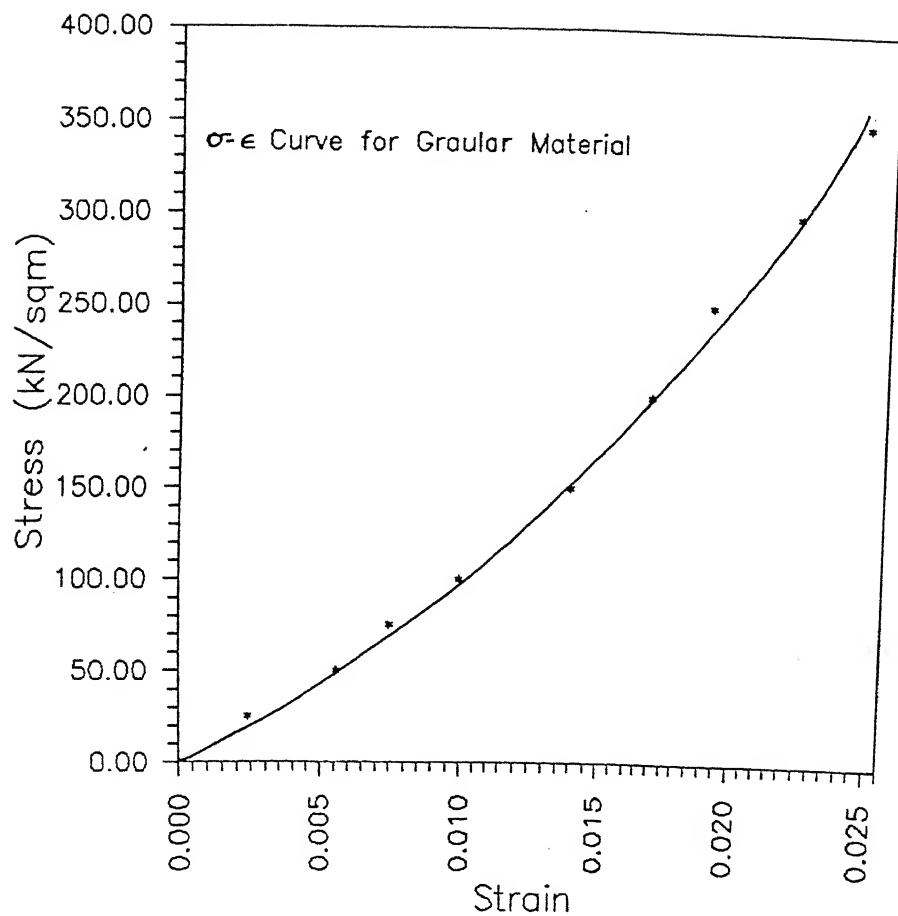


Fig.4.21 STRESS-STRAIN BEHAVIOUR OF GRANULAR MATERIAL.

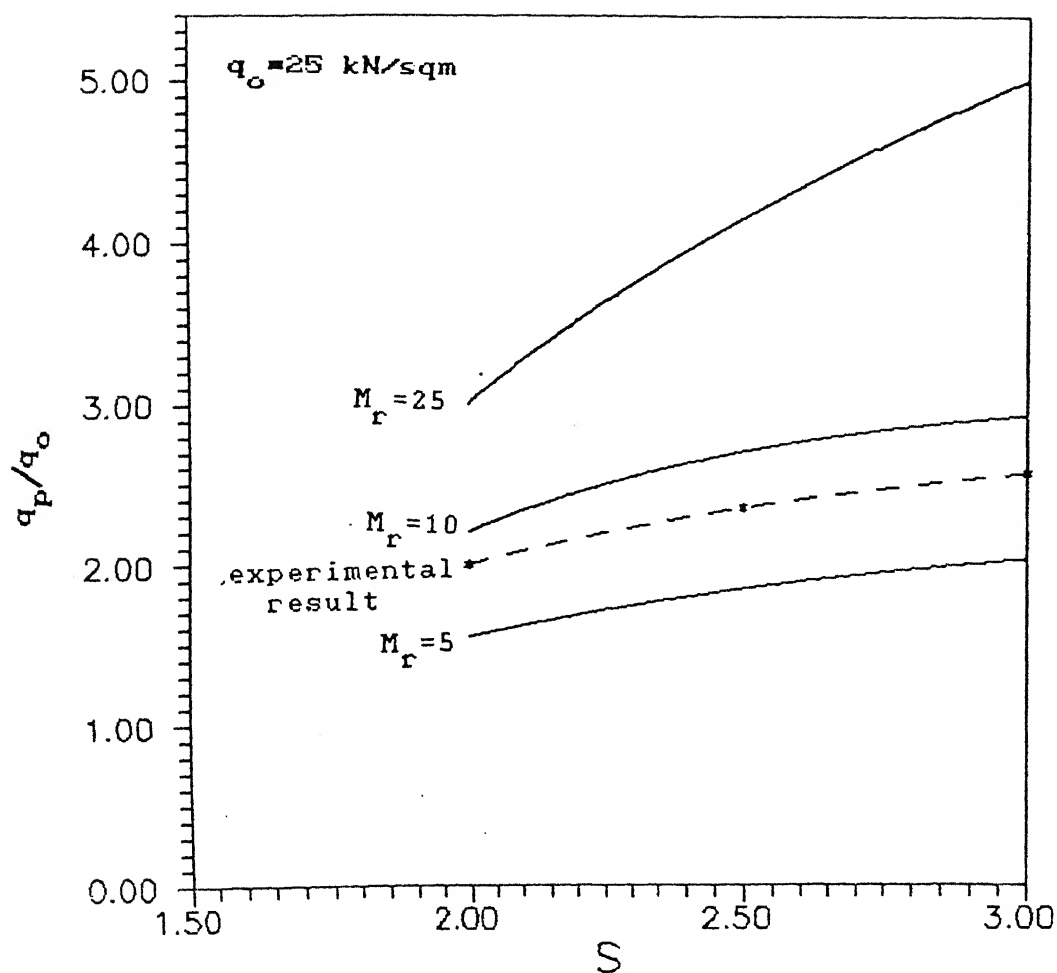


Fig.4.22 COMPARISION OF EXPERIMENTAL AND THEORETICAL RESULTS FOR THE STRESS VARIATION ON STONE COLUMN WITH COLUMN SPACING.

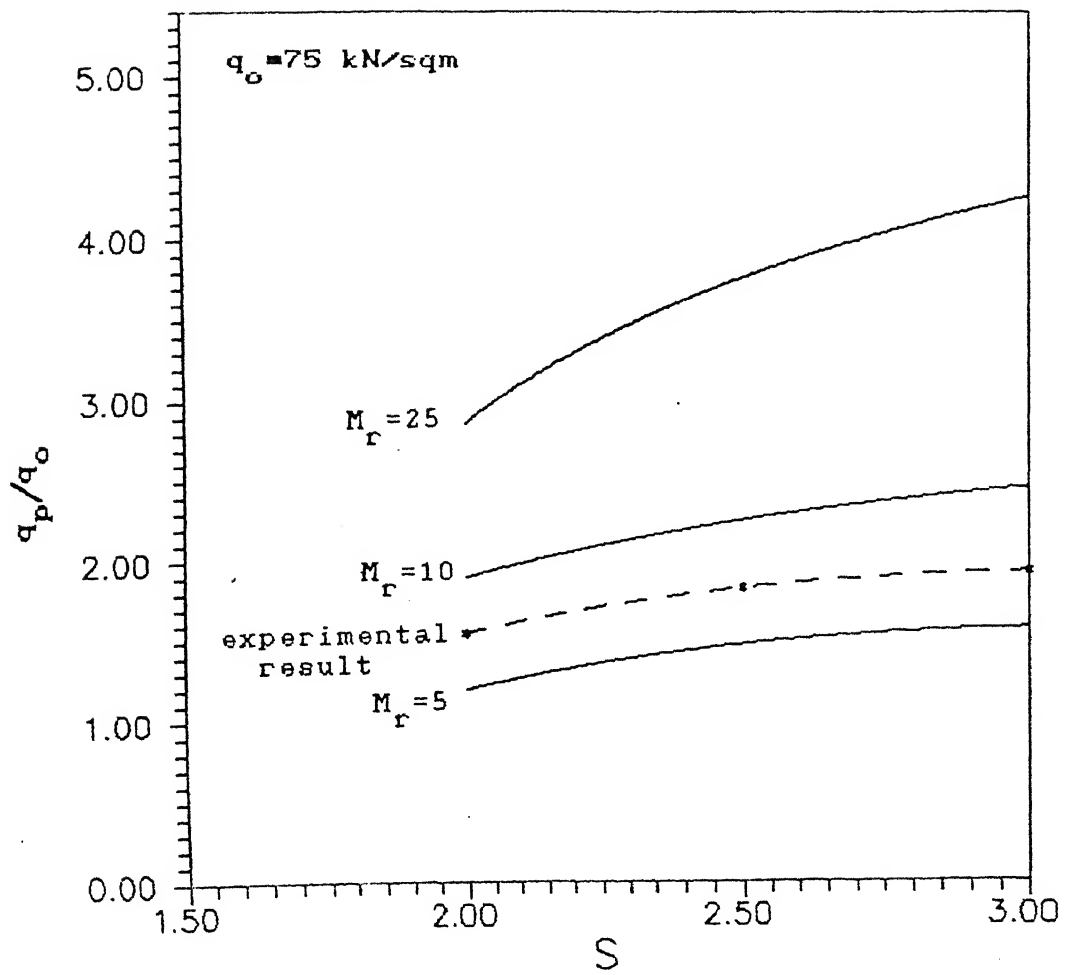


Fig.4.23 COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS FOR THE STRESS VARIATION ON STONE COLUMN WITH COLUMN SPACING.

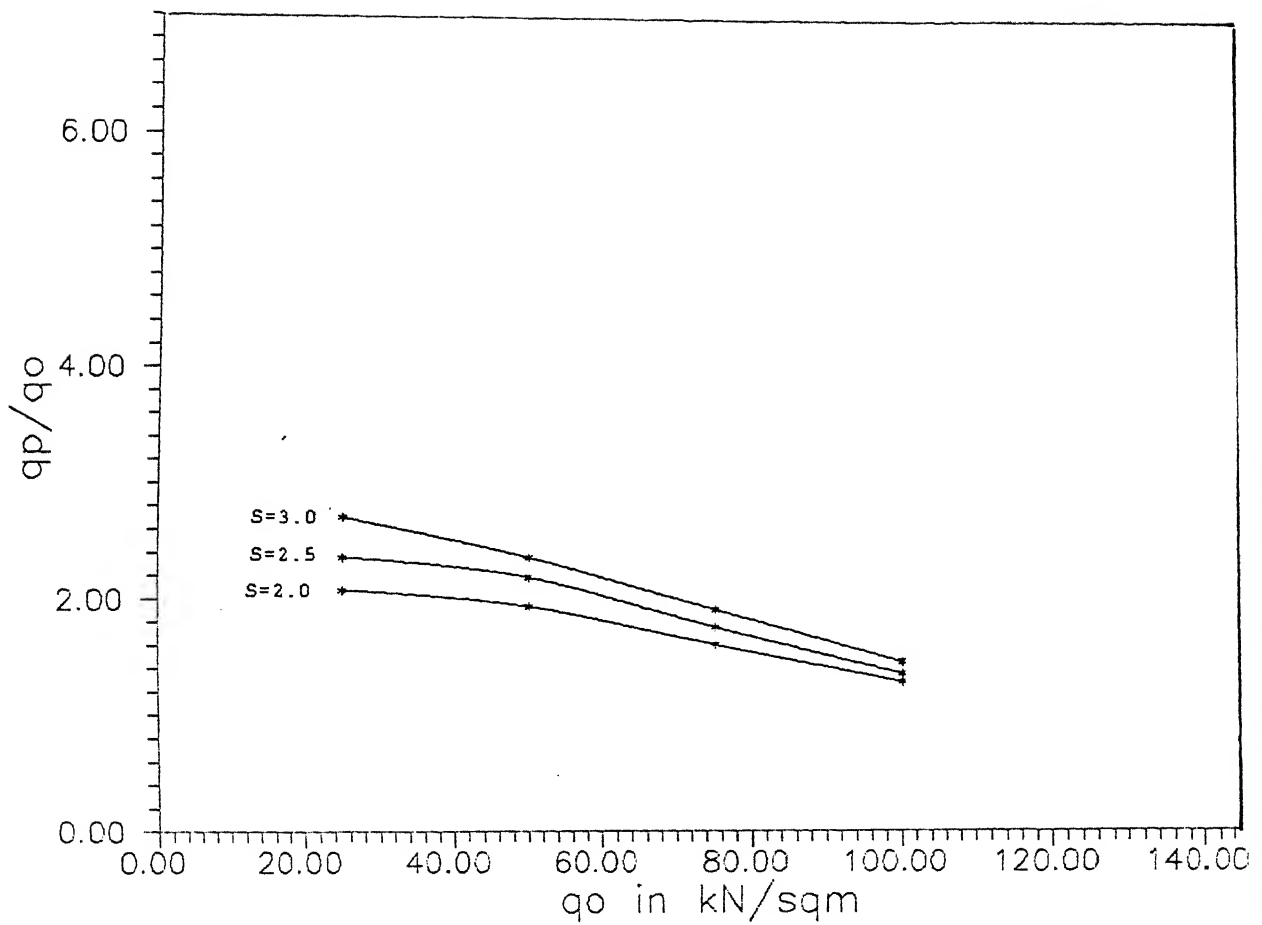


Fig.4.24 RESULTS FROM THE MODEL TESTS FOR THE VARIATION OF STRESSES ON STONE COLUMN WITH THE NORMAL STRESS.

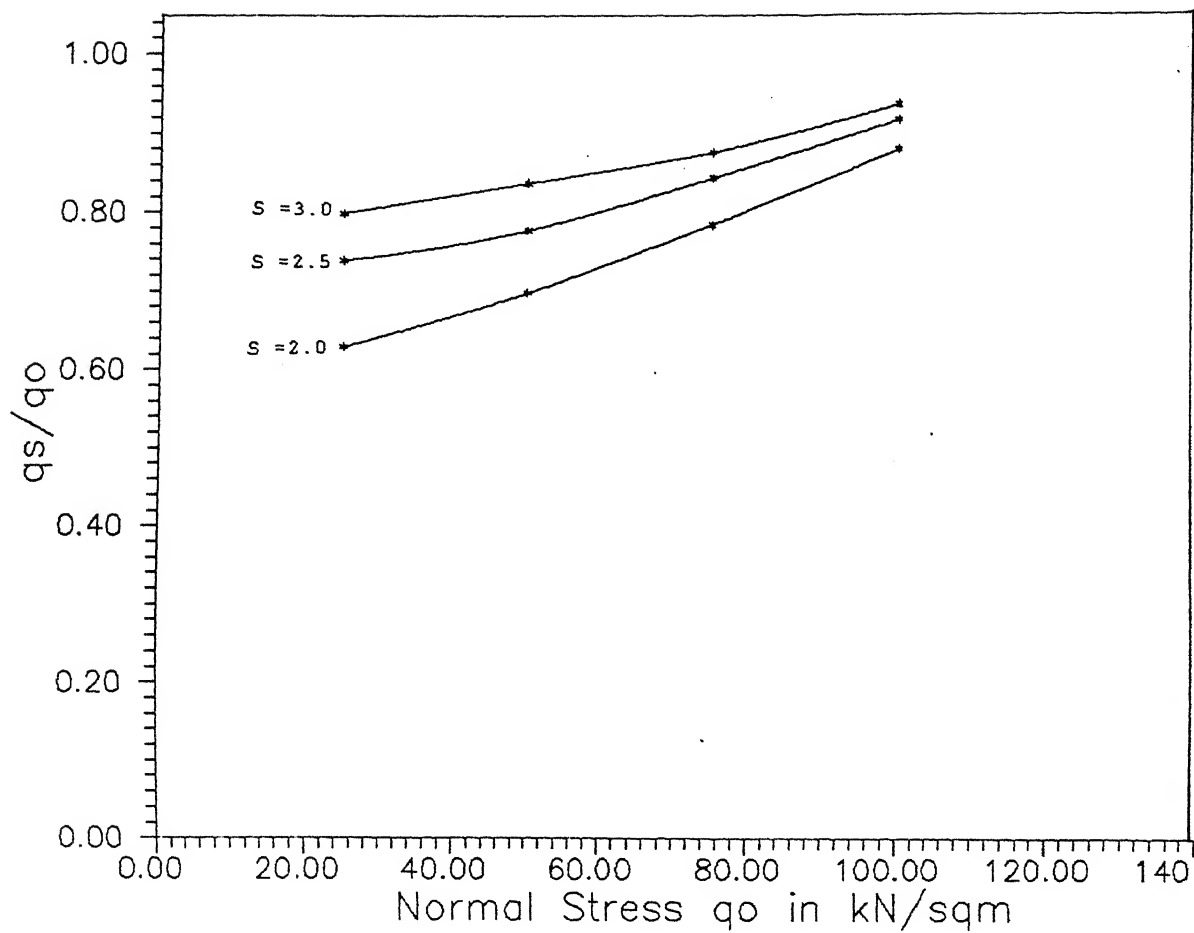


Fig.4.25 RESULTS FROM THE MODEL TESTS FOR THE VARIATION OF STRESSES ON SOIL WITH THE NORMAL STRESS.

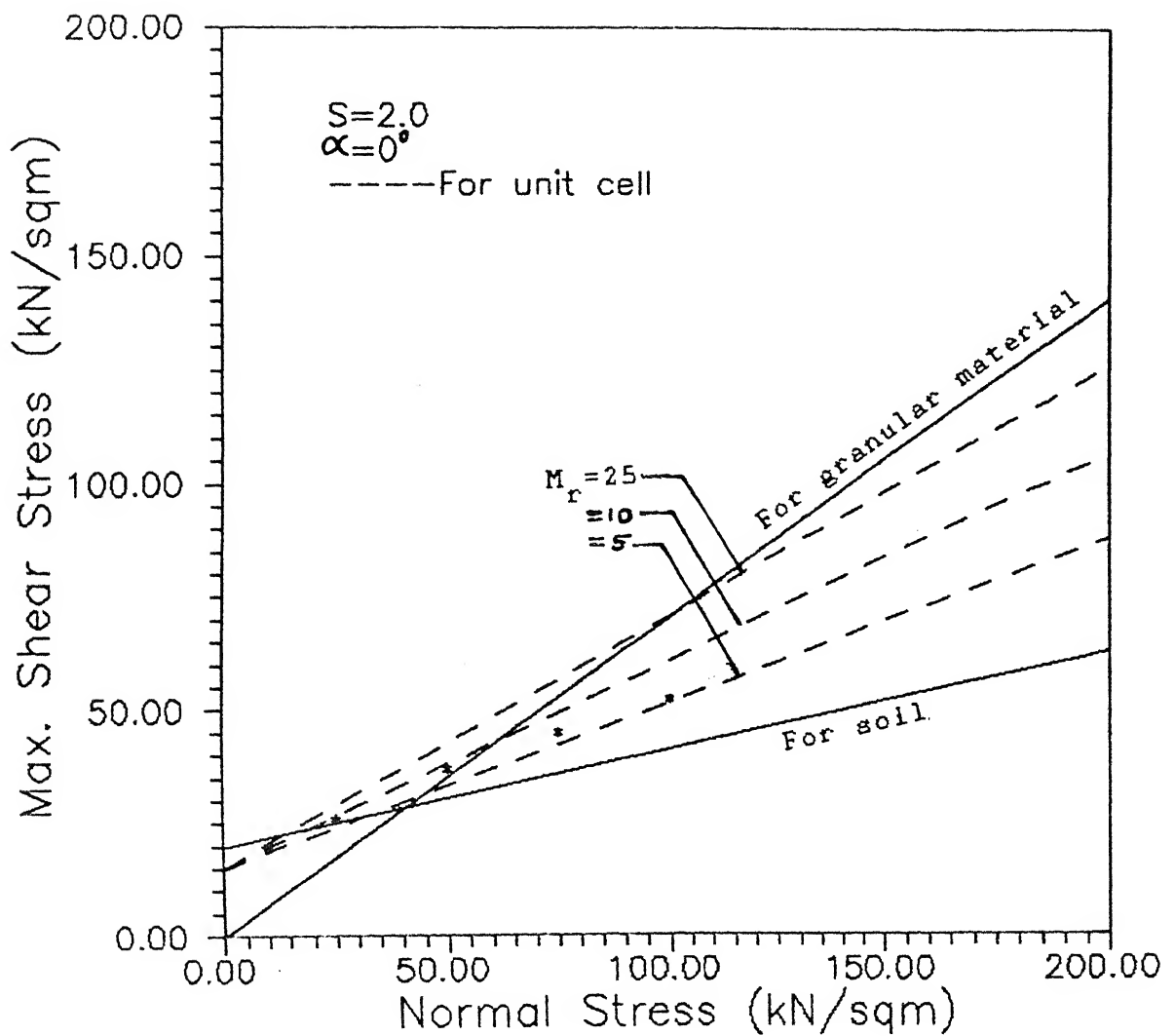


Fig.4.26 COMPARISON OF SHEAR STRENGTH OF UNIT CELL WITH STONE COLUMN AND SOIL.

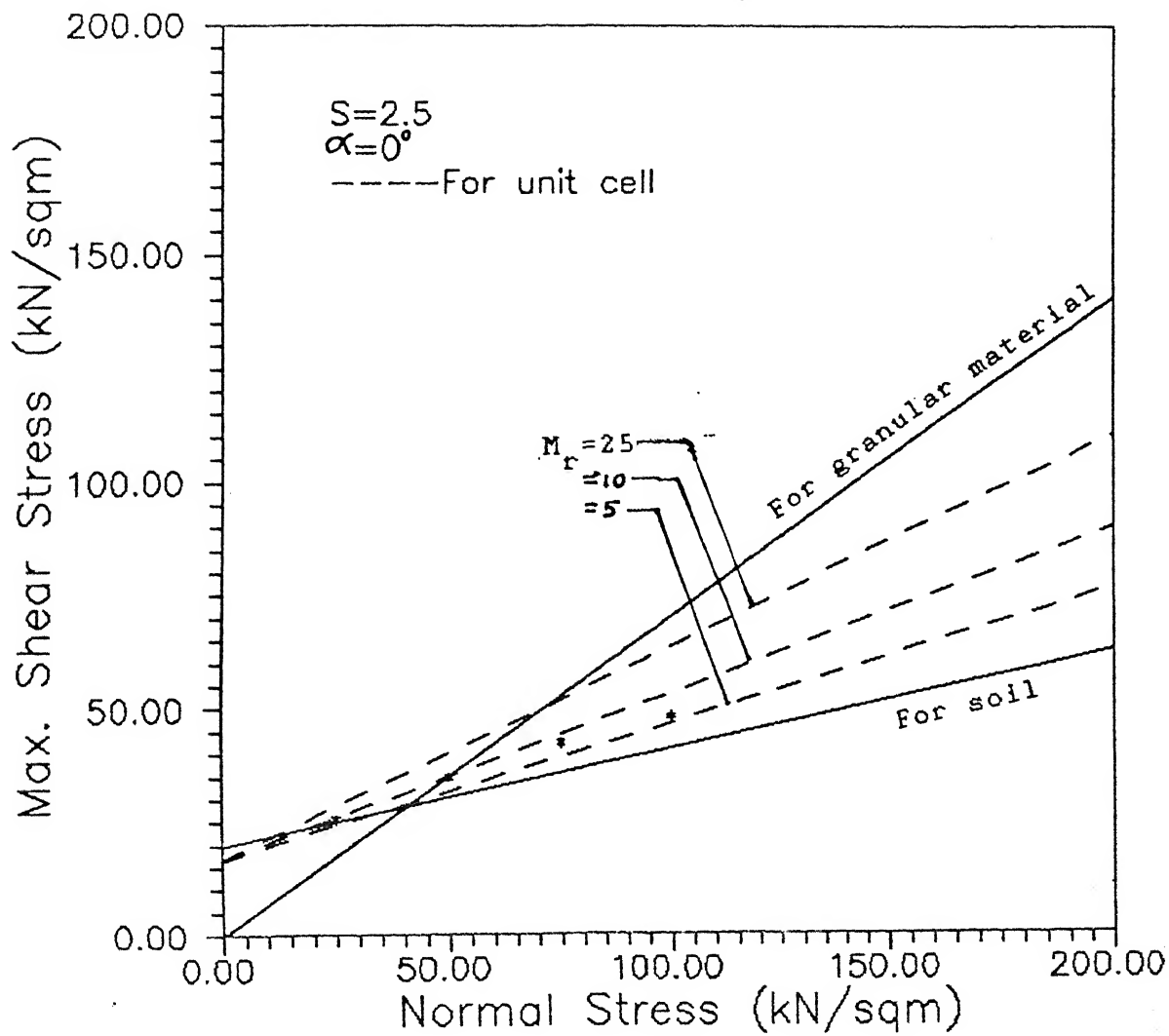


Fig.4.27 COMPARISON OF SHEAR STRENGTH OF UNIT CELL
 WITH STONE COLUMN AND SOIL.

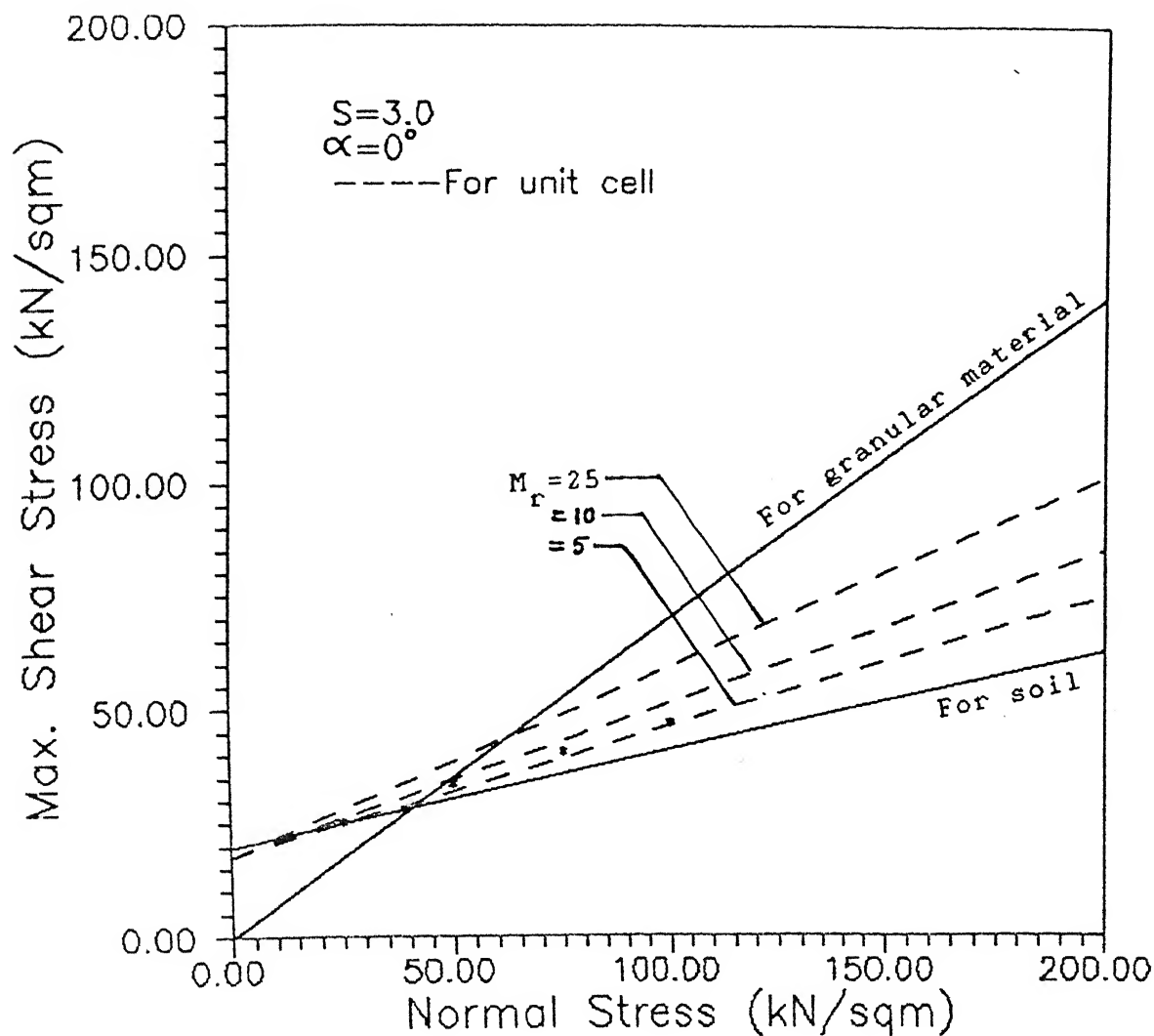


Fig.4.28 COMPARISON OF SHEAR STRENGTH OF UNIT CELL
WITH STONE COLUMN AND SOIL.

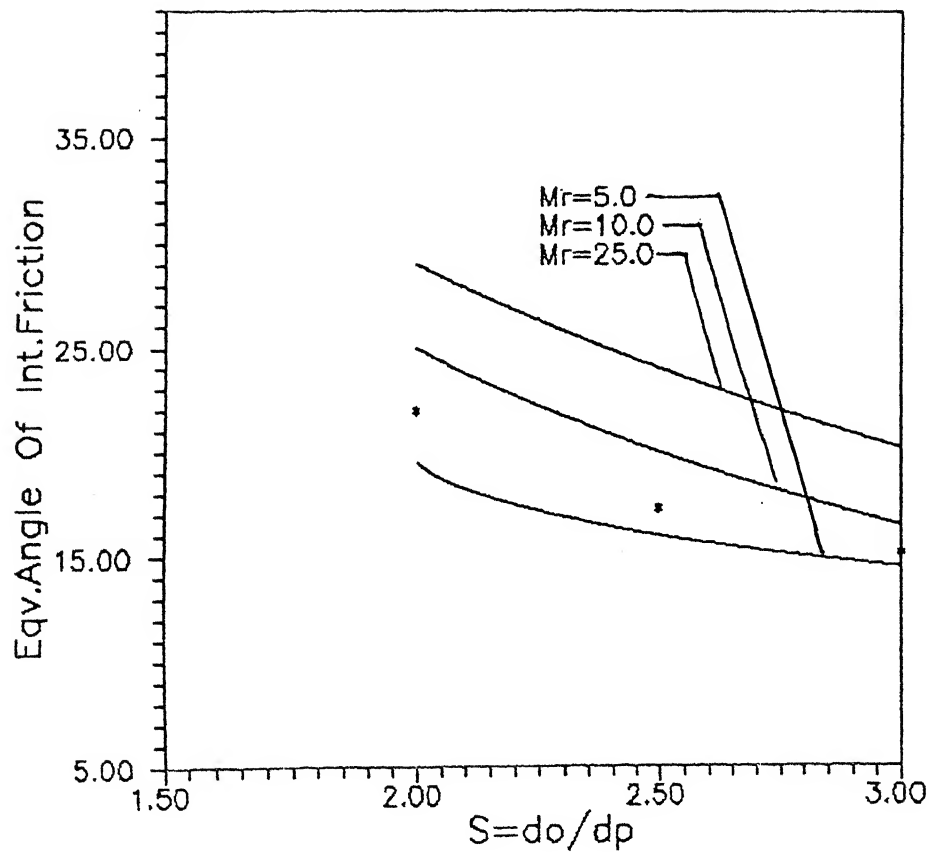


Fig.4.29 VARIATION OF EQUIVALENT ANGLE OF INTERNAL FRICTION OF UNIT CELL WITH COLUMN SPACING.

CHAPTER 5

CONCLUSIONS

1. From the analysis of a unit cell , a set of equations are obtained for the settlement of the stone column reinforced ground, and settlement reduction factor .
2. Equivalent shear strength of a unit cell can be determined from the analysis presented for a given value of normal stress, column spacing, angle of inclination of sliding surface , cohesion and angle of internal friction of soil and stone column material .
3. The analysis presented in this thesis has the advantage of analysing a non-homogeneous soil deposits as the deformation modulus of soil is considered to be vary with the depth i.e. $E_s = E_s(Z)$.
4. From the results of the theoretical and experimental analysis it is clear that a significant reduction in settlement can be achieved by the application of stone columns .
5. The analysis of unit cell reveals that when the load is applied on the unit cell then initially stress on soil may be greater than that on the stone column , but as the soil consolidates the stress on stone column becomes very high as compared to the soil .
6. With the increase in the stone column diameter the settlement reduction factor increases significantly. Further at the higher values of normal stress the settlement reduction factor

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APPENDIX A

The problem of the thick cylinder subjected to only an internal and external pressure can be solved for the case of plain stress. The stress distribution for the two dimensional problem is given below,

$$\left. \begin{aligned} \sigma_r &= \frac{r_o^2 \cdot r_p^2 (p_{oi} - p_{li})}{(r_o^2 - r_p^2) \cdot r^2} + \frac{r_p^2 \cdot p_{li} - r_o^2 \cdot p_{oi}}{(r_o^2 - r_p^2)} \\ \sigma_\theta &= \frac{r_p^2 \cdot p_{li} - r_o^2 \cdot p_{oi}}{(r_o^2 - r_p^2)} - \frac{r_o^2 \cdot r_p^2 (p_{oi} - p_{li})}{(r_o^2 - r_p^2) \cdot r^2} \\ \tau_{r\theta} &= 0 \end{aligned} \right\} \text{----- A.1}$$

@ $r = r_o$,

$$\left. \begin{aligned} \sigma_r &= -p_{oi} \\ \sigma_\theta &= \frac{2r_p^2 p_{li} - p_{oi} \cdot (r_o^2 + r_p^2)}{(r_o^2 - r_p^2)} \end{aligned} \right\} \text{----- A.2}$$

@ $r = r_p$,

$$\left. \begin{aligned} \sigma_r &= -p_{li} \\ \sigma_\theta &= \frac{p_{li} (r_o^2 + r_p^2) - 2r_o^2 p_{oi}}{(r_o^2 - r_p^2)} \end{aligned} \right\} \text{----- A.3}$$

$$\sigma_r + \sigma_\theta = \frac{2}{(r_o^2 - r_p^2)} \left\{ r_p^2 \cdot p_{li} - r_o^2 \cdot p_{oi} \right\} \text{----- A.4}$$

The three dimensional stress-strain relations in cylindrical coordinates are,

$$\left. \begin{aligned} \epsilon_r &= \frac{1}{E} \left[\sigma_r - \nu(\sigma_\theta + \sigma_z) \right] \\ \epsilon_\theta &= \frac{1}{E} \left[\sigma_\theta - \nu(\sigma_z + \sigma_r) \right] \\ \epsilon_z &= \frac{1}{E} \left[\sigma_z - \nu(\sigma_r + \sigma_\theta) \right] \end{aligned} \right\} \text{-----A.5}$$

or,

$$\sigma_z = -p_z = E \cdot \epsilon_z + \nu(\sigma_r + \sigma_\theta) \text{-----A.6}$$

$$\left. \begin{aligned} \epsilon_r &= \frac{1}{E} \left[(1-\nu^2) \cdot \sigma_r - \nu(1+\nu) \cdot \sigma_\theta \right] - \nu \epsilon_z \\ \epsilon_\theta &= \frac{1}{E} \left[(1-\nu^2) \cdot \sigma_\theta - \nu(1+\nu) \cdot \sigma_r \right] - \nu \epsilon_z \end{aligned} \right\} \text{-----A.7}$$

The strain displacement relations in cylindrical coordinates are,

$$\left. \begin{aligned} \epsilon_r &= \frac{\partial u}{\partial r} \\ \epsilon_\theta &= \frac{u}{r} + \frac{1}{r} \cdot \frac{\partial v}{\partial \theta} \\ \epsilon_z &= \frac{\partial w}{\partial z} \end{aligned} \right\} \text{-----A.8}$$

Substituting equation A.1 into A.6 and A.7.

$$\left. \begin{aligned} \epsilon_z &= \frac{-p_z}{E} - \frac{2\nu}{E} \left[\frac{p_{oi} r_p^2 - p_{oi} r_o^2}{(r_o^2 - r_p^2)} \right] \\ \epsilon_r &= \frac{(1+\nu)}{E} \left[\frac{p_{li} r_p^2 - p_{oi} r_o^2}{(r_o^2 - r_p^2)} \cdot (1-2\nu) + \frac{r_o r_p (p_{oi} - p_{li})}{(r_o^2 - r_p^2) \cdot r^2} \right] - \nu \epsilon_z \end{aligned} \right\} \text{A.9}$$

$$\epsilon_{\theta} = \frac{(1+\nu)}{E} \left[\frac{p_{1i} r_p^2 - p_{oi} r_o^2}{(r_o^2 - r_p^2)} \cdot (1-2\nu) - \frac{r_o^2 r_p^2 (p_{oi} - p_{1i})}{(r_o^2 - r_p^2) \cdot r^2} \right] - \nu \epsilon_z \quad \text{A.10}$$

Combining equation A.8 and A.9, gives,

$$\frac{\partial u}{\partial r} = \frac{(1+\nu)}{E} \left[\frac{p_{1i} r_p^2 - p_{oi} r_o^2}{(r_o^2 - r_p^2)} \cdot (1-2\nu) + \frac{r_o^2 r_p^2 (p_{oi} - p_{1i})}{(r_o^2 - r_p^2) \cdot r^2} \right] - \nu \epsilon_z \quad \text{A.11}$$

$$\frac{u}{r} + \frac{1}{r} \cdot \frac{\partial u}{\partial r} = \frac{(1+\nu)}{E} \left[\frac{p_{1i} r_p^2 - p_{oi} r_o^2}{(r_o^2 - r_p^2)} \cdot (1-2\nu) + \frac{r_o^2 r_p^2 (p_{oi} - p_{1i})}{(r_o^2 - r_p^2) \cdot r^2} \right] - \nu \epsilon_z \quad \text{A.12}$$

$$\frac{\partial w}{\partial z} = - \frac{p_z}{E} - \frac{2\nu}{E} \left[\frac{p_{oi} r_p^2 - p_{oi} r_o^2}{(r_o^2 - r_p^2)} \right] \quad \text{A.13}$$

Integrating equation A.11 ,

$$u = \frac{(1+\nu)}{E} \left[\frac{p_{1i} r_o^2 - p_{oi} r_o^2}{(r_o^2 - r_p^2)} (1-2\nu) \cdot r - \frac{r_o^2 r_p^2 (p_{oi} - p_{1i})}{(r_o^2 - r_p^2) \cdot r} \right] - \nu \epsilon_z r \quad \text{A.14}$$

APPENDIX B

The ratio of the settlement of stone column treated area to the untreated area can be calculated without considering the radial stresses and therefore the problem becomes one dimensional. The one dimensional analysis may be carried out as follows,

$$\beta = \frac{\text{Settlement of treated ground } (\epsilon_z)_t}{\text{Settlement of untreated ground } (\epsilon_z)_{ut}}$$

From normal strain compatibility,

$$\frac{q_p}{q_s} = \frac{E_p}{E_s} = M_r \quad \text{..... B.1}$$

Equilibrium requires,

$$q_o \cdot r_o^2 = q_p \cdot r_p^2 + q_s (r_o^2 - r_p^2)$$

$$q_o = q_p \cdot a_p/a_o + q_s (1 - a_p/a_o)$$

Where, a_p/a_o = Area ratio = $1/S^2$

$$q_o = q_p / S^2 + q_s (1 - 1/S^2) \quad \text{..... B.2}$$

From Eqⁿ B.1 and B.2,

$$q_p = \frac{q_o \cdot E_p}{1/S^2 \cdot E_p + (1 - 1/S^2)} = q_o \cdot E_p / E_{eq} \quad \text{..... B.3}$$

$$q_s = \frac{q_o \cdot E_s}{1/S^2 \cdot E_p + (1 - 1/S^2)} = q_o \cdot E_s / E_{eq} \quad \text{..... B.4}$$

Where,

$$E_{eq} = 1/S^2 \cdot E_p + (1 - 1/S^2) E_s \quad \text{..... B.5}$$

Now,

$$(\epsilon_z)_t = q_o / E_{eq} = \frac{q_o}{1/S^2 \cdot E_p + (1 - 1/S^2)}$$

and

$$(\epsilon_z)_{ut} = q_o / E_s$$

therefore,

$$\beta = \frac{E_s}{1/S^2 \cdot E_p + (1 - 1/S^2)}$$

$$\beta = \frac{1}{1 + (E_p / E_s - 1) / S^2}$$

$$\beta = \frac{1}{1 + (M_r - 1) / S^2} \quad \text{..... B.6}$$